Transport and Stability Properties of Quasi-Omnigeneous, Low Aspect Ratio, Hybrid Stellarators

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There is great potential for improving the attractiveness of stellarator reactors if the unit size can be substantially reduced. A difficulty with designing low aspect ratio stellarator systems has been the tendency for both β limits and $1/\nu$ regime transport to degrade with decreasing aspect ratio, A. However, recently two approaches have been developed to reduce or eliminate ripple transport in small aspect ratio systems through plasma shaping: (1) by imposing an approximate symmetry of the magnetic field spectrum in Boozer coordinates (quasi-helical, quasi-axisymmetric), or (2) by requiring that bounce average drift orbits approximately align with flux surfaces (quasi-omnigeneity) [1]. In contrast to the approach taken in the design of W7-AS and W7-X, we do not require complete cancellation of the net bootstrap current. The resulting current allows for increased β and additional control over the rotational transform profile. The design objective is to employ a substantially lower net current than in a comparable tokamak (< 30%) so as not to reintroduce the danger of plasma disruption. Quasi-omnigeneous configurations are obtained by a non-linear optimization procedure in which the plasma shape and current profile are adjusted in order to achieve certain targeted physics and engineering objectives. Once the configuration is obtained, a set of modular coils is "reverse engineered". Using these techniques we have obtained attractive configurations for a modest "Proof of Principle" scale device, (R_0 ~ 1.5 m, B_0 ~ 1.4 T, $A \sim R_0/\langle a \rangle = 3.6$), and for a small "Concept Exploration" scale device, (R_0 ~ 1.0 m, B_0 ~ 1.0 T, $A \sim R_0/\langle a \rangle = 4$). Preliminary examination of reactor scale devices show that acceptable alpha particle confinement can be obtained.

Optimization procedure

Great simplification in calculating and understanding confinement in 3D systems can be achieved by working in a particular flux coordinate system known as Boozer coordinates [2]. In this system, and in the absence of electric field, the drift orbit Hamiltonian depends on space only through the magnitude of the magnetic field, $|B|$. Any symmetry in $|B|$, therefore, yields a constant of motion which assures good particle confinement. Systems having a single dominant helicity in the Fourier spectrum of $|B|$ are referred to as quasi-symmetric and tend to have very good neoclassical confinement [3,4]. However it has recently been shown that magnetic symmetry is sufficient, but not necessary, for good confinement [1,5]. Symmetric configurations are special cases of a more general criterion

known as omnigeneity, such that bounce average particle drift orbits do not deviate from flux surfaces $\langle \mathbf{v}_D \quad \psi \rangle = \partial J / \partial \theta = 0$, where J is the second adiabatic invariant. We have developed a method of confinement optimization in which contours of an approximate second adiabatic invariant, J^*, are made to align with flux surfaces [6,7]

$$J^*(\psi,\theta) = \int_{\psi,\theta=const} \frac{g(\psi)d\varphi}{B} m|v_{\parallel}| - \sigma_{\parallel} \frac{2\pi}{N} \psi_p,$$

with

$$\sigma_{\parallel} = \begin{cases} +1, & \text{copassing orbits,} \\ 0, & \text{locally trapped orbits } (v_{\parallel} = 0 \text{ on orbit}), \\ -1, & \text{counterpassing orbits,} \end{cases}$$

where

$$v_{\parallel} = \pm \left(\frac{2}{m}(\varepsilon - \mu B - q\Phi)\right)^{1/2}, \quad g(\psi) = \text{poloidal current}, \quad \psi_p = \text{poloidal magnetic flux}$$

function, N = number of field periods, ε = total energy, μ = magnetic moment, q = charge, and m = mass. This is a very flexible criterion and we find that it is possible to maintain good confinement properties while controlling other desirable physics properties such as stability, and bootstrap current profile.

A new optimization code has been developed which uses a nonlinear Levenberg-Marquardt scheme to minimize an objective function χ^2 representing the various physics or engineering targets with adjustable weights σ [8,9],

$$\chi^2 = \frac{1}{\sigma \langle J^* \rangle} \left\langle \left(\frac{\partial J^*}{\partial \theta} \right)^2 \right\rangle + \ldots$$

Typically the targets for confinement optimization include: alignment of $B_{min,}$ with ψ to improve confinement of deeply trapped particles, alignment of B_{max} with ψ to minimize the number of transitional particles, alignment of J^* with ψ for a range of pitch angles to better confine the whole population of trapped particles, and minimization of magnetic ripple, $B_{max} - B_{min}$ to reduce the number of trapped particles. Other targets include: specified iota profile $\iota(\psi)$, specified aspect ratio R_0/a, magnetic well V'' < 0, upper bound on plasma current $I_P < I_{max}$, and limit on local magnetic surface curvature to avoid excessive elongation or cusps. The control variables for the optimization consist of the Fourier harmonics of the outermost flux surface shape (typically about 20 are used) and expansion coefficients of the current density profile.

In operation an initial configuration is selected from a previously computed VMEC equilibrium, either from simple coils or from a previous optimization. Iteration is performed under control of the Levenberg-Marquardt optimizer in which the boundary shape and current profile are adjusted, a new VMEC equilibrium is calculated, the equilibrium is converted to Boozer coordinates, and the objective function χ^2 is computed. Detailed assessment of the final optimized configuration is carried out by post processing with confinement (Monte Carlo and orbit codes), stability, bootstrap, and coil analysis codes.

Optimization Studies

Initial optimization studies focused on "Proof of Principle" scale devices of modest size (R_0 ~ 1.5 m, B_0 ~ 1.5 T) but having sufficient performance to definitively address the key physics issues: confinement optimization through quasi-omnigeneity, attractive β ($\langle\beta\rangle$ > 5%), and control of bootstrap current profile. Configurations having different numbers of field periods (2 to 8) were studied. It was easier to obtain high β limits with the higher field period numbers but these configurations required coils relatively close to the plasma in order to obtain the $|B|$ variation needed for omnigeneity. Now the most attractive configurations appear to be the lower field period numbers (2 to 4). Figure 1 shows a 3D plot of the last closed flux surface of a 4 field period configuration having $R_0/\langle a\rangle$ = 3.6. Figures 2a and 2b show the flux surfaces at the $\varphi = 0°$ and $\varphi = 45°$ planes for $\langle\beta\rangle$ = 6.75%. The ι profile is monotonically increasing from ι = 0.13 at the center to ι = 0.38 at the edge. Strong shear exists at the outer edge, which is favorable for stability. This configuration was found to be Mercier, ballooning, and kink stable at this β. At the time of this work, we were not able to extract coils for finite β configurations, so a fixed boundary equilibrium was calculated having the same shape as Figs. 2a and 2b, but with at β = 0, I_P = 0. A very modest Shafranov shift occurred and there was a slight increase in $\iota_{edge} - \iota_0$ as β changed from zero to 7%. A set of modular coils was generated to produce this vacuum configuration, which are also shown in Fig. 1. Using these coils to calculate a free boundary equilibrium we recovered the original fixed boundary, β = 0 flux surfaces with good accuracy. Figures 2c and 2d show that Poincare plots of the vacuum fields produced by the modular coils set are again in good agreement with the free boundary equilibrium. One can see small island chains in the Figs. 2c and 2d, but these are not thought to be a serious difficulty. We have not as yet obtained a complete coil set which can maintain the optimized plasma shape as β is increased.

We used a Monte-Carlo model [10] having pitch angle and energy scattering to estimate the transport properties of this configuration assuming plasma parameters: $T_i = T_e$ = 1.0 keV, n_e = 5.0 x 10^{19} m^{-3}, I_P = 120 kA. The test particle energy is taken as $2T_I$ and a parabolic ambipolar potential well is included with $e\varphi / T_I$ = 1. The global confinement time is estimated to be about τ_E = 50 msec, which is about 7 times the empirical scaling of ISS95. Post optimization calculation of the bootstrap current gave about twice the value assumed in

the equilibrium. We have shown that considerable control of I_{BS} is possible by targeting various harmonics of the Boozer spectrum. We have plans to include the bootstrap current within the optimization procedure so that self-consistency can be maintained.

More recently we have been examining the properties of smaller, "Concept Exploration" scale devices, R_0 = 1 m, B_0 = 1T, assuming relatively modest heating power (~0.4 MW of ECH and ~1 MW of ICRH). The goals of such a device would be to study transport and confinement optimization by quasi-omnigeneity, effects of radial electric fields, confinement of energetic particles from ICRH, turbulence properties and stability at moderate β. The critical optimization objectives are achievement of good neoclassical confinement and confinement of deeply trapped energetic ions as would be expected from ICRH. We have considered 3 and 4 field period configurations having A in the range 3.6 to 4.2, each with two types of iota profile: (1) low iota ($0.25 \leq \iota(\psi) \leq \sim 0.45$), and (2) high iota ($0.68 \leq \iota(\psi) \leq \sim 0.79$). Figure 3 shows the Monte Carlo calculation of the energy loss rate through the last closed flux surface for the above four configurations, assuming B_0 = 1 T, test particle energy = 2 keV, background plasma density = 5 x 10^{19} m^{-3}, plasma temperature = 1 keV. The plasma potential is taken to be zero. Also shown in Fig. 3 are similar results for ATF (which has similar minor radius but R_0 = 2.1 m) and also the W7-X configuration scaled to the same minor radius a = 0.28 m but proportionately larger major radius. We see that both high iota cases and the N = 3 low iota case have about twice the confinement time of ATF (at half the major radius) and about half the confinement time of the W7-X configuration, but at one third the aspect ratio. Investigation of the high-n ballooning stability gives β limits for these configurations in the range of <β> = 0.5% to 3% depending on the width of the pressure profile, the N = 4 case yielding the higher β values.

Conclusions

We have developed a new optimization procedure for low aspect ratio stellarators that targets bounce-average omnigeneity by aligning contours of constant J* with magnetic flux surfaces. We have found interesting configurations having good stability and transport which are neither quasitoroidal nor quashi-helical. The optimization procedure is quite flexible so that additional target functions can easily be included. We are presently in the process of making ballooning stability and consistency of the bootstrap current direct targets of the optimization. Soon, we expect to propose a small "Concept Exploration" type experiment to begin testing these principles.

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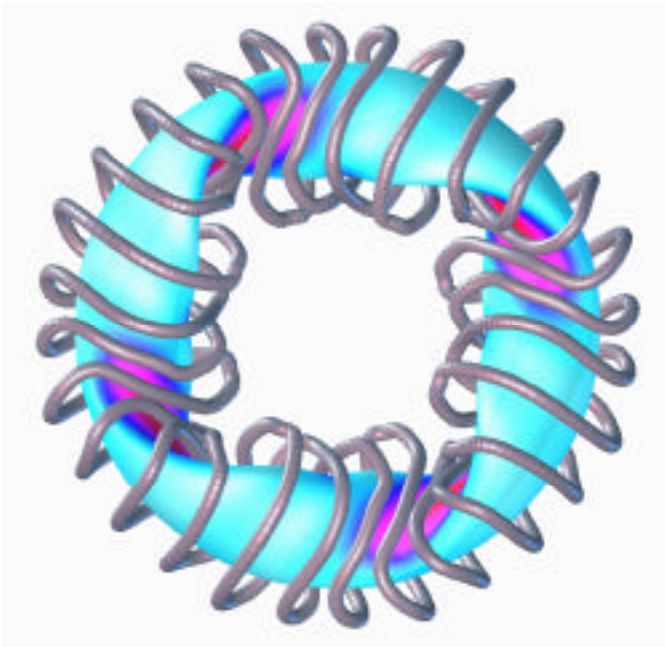

Fig. 1. Outermost flux surface and modular coils for a 4 field period Quasi-omnigeneous hybrid stellarator

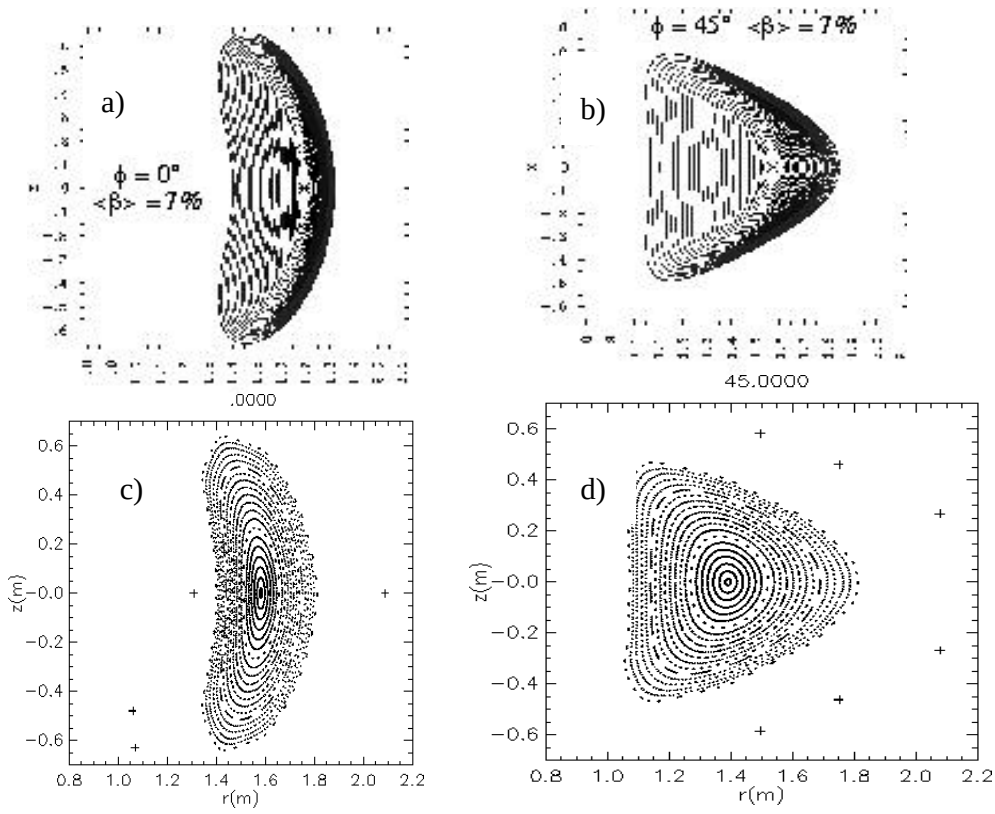

Fig. 2. Flux surfaces at $\varphi = 0$ and half period toroidal planes (a, b) for $\beta = 7\%$, and associated Poincare plots for B field from modular coils (c,d)

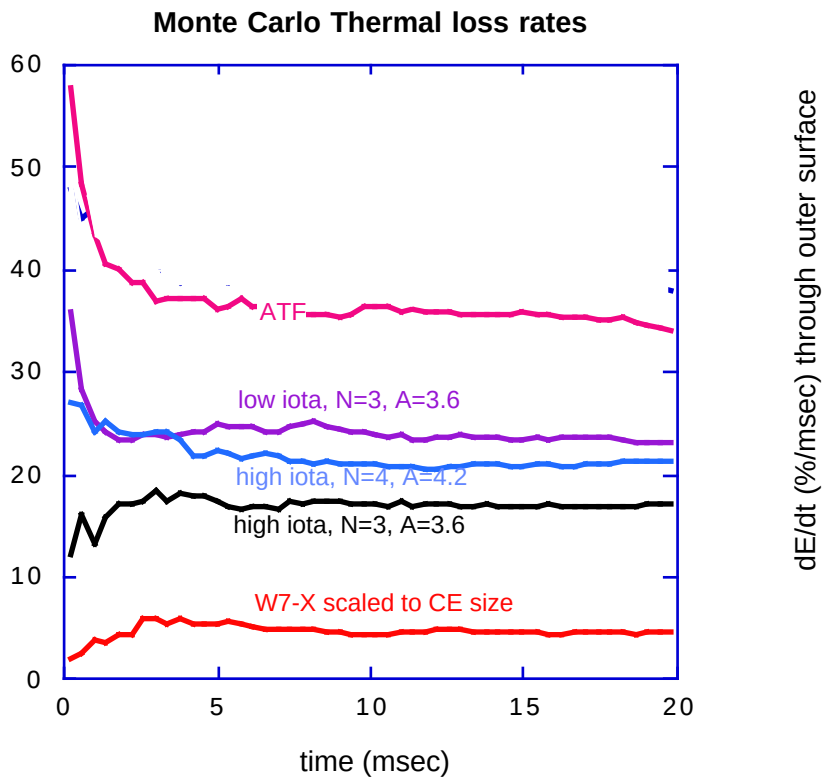

Fig. 3. Monte Carlo calculation of energy loss rate out the last closed flux surface

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