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QPS PLASMA AND COIL OPTIMIZATION

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Abstract. Compact stellarators offer the potential of both attractive fusion reactors as well as lower cost near term experiments. The optimization of compact stellarators is a challenging physics and computational problem whose solution has required the development of new tools. These include: a variety of rapidly evaluated transport and 3D ballooning targets, a bootstrap current consistency condition, and an upgraded version of the VMEC equilibrium code with improved convergence; these components are immersed in an optimization algorithm that has been adapted to utilize multiple processor parallel computers. Following the physics optimization of the outer flux surface, a set of coils is developed (COILOPT code) by directly adjusting the coil geometry and currents so as to minimize the normal component of magnetic field on the outer flux surface in conjunction with coil engineering constraints. Based on these methods, new stellarator hybrid configurations have been developed with quasi-poloidal symmetry at very low aspect ratios ($\langle R\rangle/\langle a\rangle \sim 2.7$) that achieve low levels of neoclassical transport and good stability properties, including access to high β second stability states.

I. Introduction

The QPS[1] (Quasi Poloidal Stellarator) is a compact (A = 2.7) two field period quasi–poloidally symmetric device that has been developed through a systematic optimization procedure that balances physics requirements (confinement quality, plasma stability to pressure and current-driven modes, bootstrap current consistency), design goals (low aspect ratio, minimum plasma width for good neutral shielding, avoidance of low order rational surfaces, rotational transform provided predominantly from coils and engineering constraints (adequate coil-plasma and coil-coil separation, sufficient space in the center for an Ohmic heating transformer and toroidal coils). The successful optimization of low aspect ratio devices has been possible due to a number of past scientific advances: (a) the identification of an appropriate coordinate system[2] within which magnetic field symmetries leading to improved particle orbit confinement could readily be identified; (b) demonstration that numerical optimizations of three-dimensional systems could lead to good equilibrium, transport, and stability properties[3]; (c) numerical design[4] of coils to accurately produce these configurations; and (d) the increasing availability of massively parallel computers with capacities of 1 teraflop and higher coupled with the development of efficient algorithms to take advantage of this resource.

II. Plasma Optimization

Our optimization technique, known as the STELLOPT code, has evolved to include a range of physics and engineering targets; these are summarized in table 1.

Targets (Physics/Engineering)	Example
Bounce-average omnigeneity (drift surfaces and flux surfaces aligned)	$B_{min} = B_{min}(\psi)$, $B_{max} = B_{max}(\psi)$, $J = J(\psi)$
Target nearby quasi-symmetries	Minimize B_{mn} if $m \neq 0$ (QP), or if $m/n \neq 1$ (QH)
Local diffusive transport	D, χ from DKES, $\varepsilon_{eff}^{3/2}$ from NEO[5] code
Current profile, bootstrap consistency	self-consistent I_{BS}, $I(\psi)$ goes to 0 at edge
Limit maximum plasma current	e.g., $I_{max} < 80$ kAmps
Iota profile	$i(\psi) = 0.3$ ($\rho = 0$) 0.4 ($\rho = a$)
Magnetic Well, Mercier	$V'' < 0$, $D_M > 0$ over cross section
Ballooning stability	$\langle\beta\rangle \sim 2$-4%
Aspect ratio	$R_0/a \approx 2.5$ to 3.5
NESCOIL targets/feasible coil design	Complexity, B_{err}, Max. current density
Adequate shielding of neutrals	Minimum "waist" thickness
Fit within vacuum tank and allow room for Ohmic transformer	$R_{max} < 1.5$ meter, $R_{min,\ inboard} > 0.4$ meter
Limit outer surface curvature	avoid strong elongation/cusps

Table 1 - Physics and engineering targets used in the optimization procedure

The QPS transport optimizations initially focused on improving both the omnigeneity measures listed above and DKES transport coefficients evaluated near the plateau regime on 4-5 flux surfaces. At lower aspect ratios (in the range A = 2.5 – 3), it became clear from the variation of |B| that these targets were driving the optimization towards configurations with a dominant quasi-poloidal symmetry [i.e., $|B| = |B|(\zeta)$ where ζ =toroidal angle in Boozer coordinates]. Although subsequent optimizations targeted quasi-poloidal symmetry more directly, some remaining residual degree of non-symmetry is generally present. By adjusting the form of this residual, using transport targets such as DKES and $\varepsilon_{eff}^{3/2}$ from the NEO[5] code further gains in confinement have been possible. We now have configurations with a factor of 3 – 4 lower effective ripple ε_{eff} as measured by the NEO[5] code than the original QP configuration that was analyzed in ref. 1; also, by another simple measure, the ratio of magnetic energy in the non quasi-poloidal modes to that in the symmetric modes, the recent configurations are improved by a factor of ~2 as compared to the configuration of Ref. 1.

Ballooning stability has also been targeted in QPS optimizations using the COBRA[6] code. This provides a rapid, but accurate evaluation based on VMEC coordinates of 3D ballooning stability. The ballooning growth rates are targeted on multiple flux surfaces and can be reduced by the optimizer either through changes to the outer flux surface shape or via plasma pressure profile modifications. Our configurations typically have first stability ballooning limits of $\langle\beta\rangle \sim 2\%$. In addition, QPS devices have second stability regimes to ballooning modes; these regimes are generally present when the outward equilibrium flux surface shift is significant, leading to enhanced poloidal fields and shorter connection lengths on the outboard side of the plasma. Near term QPS configurations, with stellarator-like shear, have second stability regimes, provided the bootstrap current can be suppressed from collisionless levels. QPS devices with

tokamak-like shear,[7] which are being considered as reactor candidates, also have such regimes; ballooning and kink stable cases have been found for <β>'s up to 15%. Although the access to these high <β> regimes must currently pass through a window of instability, future work will be directed towards finding stable access paths.

The plasma optimization targets of Table 1 are controlled by a Levenberg-Marquardt (LM) optimization algorithm. In order to take advantage of massively parallel architectures, we have developed a global parallelization method for LM. This is carried out both in the periodic Jacobian evaluations (over the 30 – 60 dimensional parameter spaces STELLOPT typically operates in) and in the estimation of the Levenberg parameter. This high level parallelization over variations in the independent variables (shape coefficients and profile parameters) simplifies the development of modules used to calculate the targets listed in Table 1. A bank-queuing algorithm is used in distributing the computational tasks to the processors to accommodate for the fact that they are uneven in length. Close to linear speed-ups are observed up to the point where the number of processors equals the number of independent variables. This scaling with the number of processors is indicated in Figure 1.

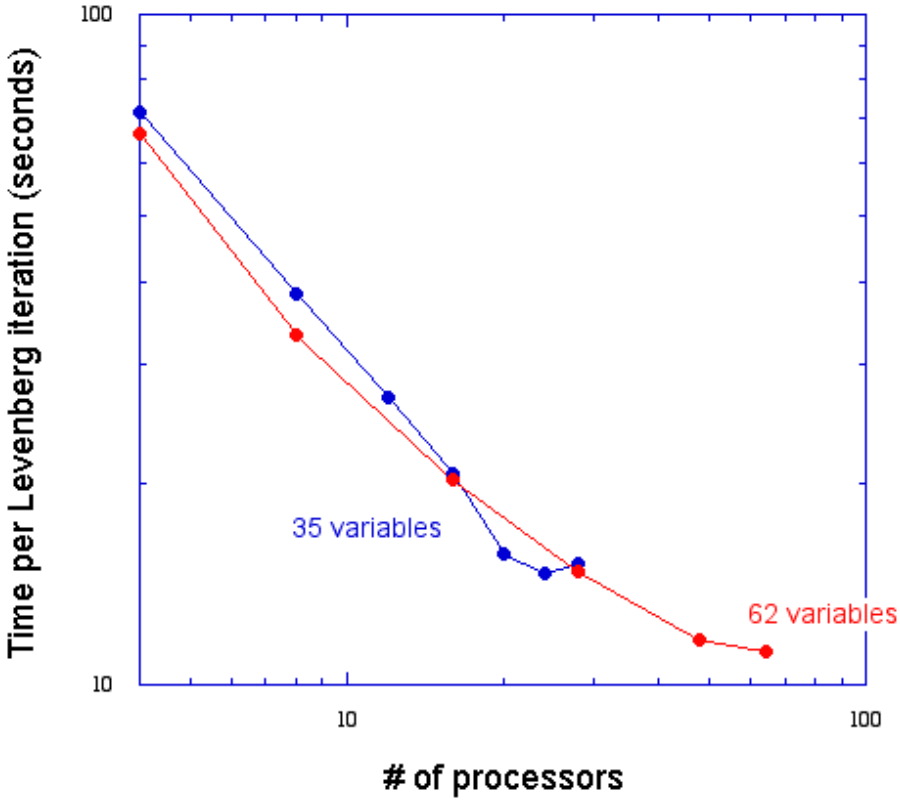

Fig. 1 – Scaling of time per Levenberg-Marquardt optimization step with number of processors for test cases with 35 and 62 independent variables.

In Figure 2 the modular coil geometry and flux surface shapes are displayed for a recent QPS design arising from the above optimization process. There are eight coils per field period

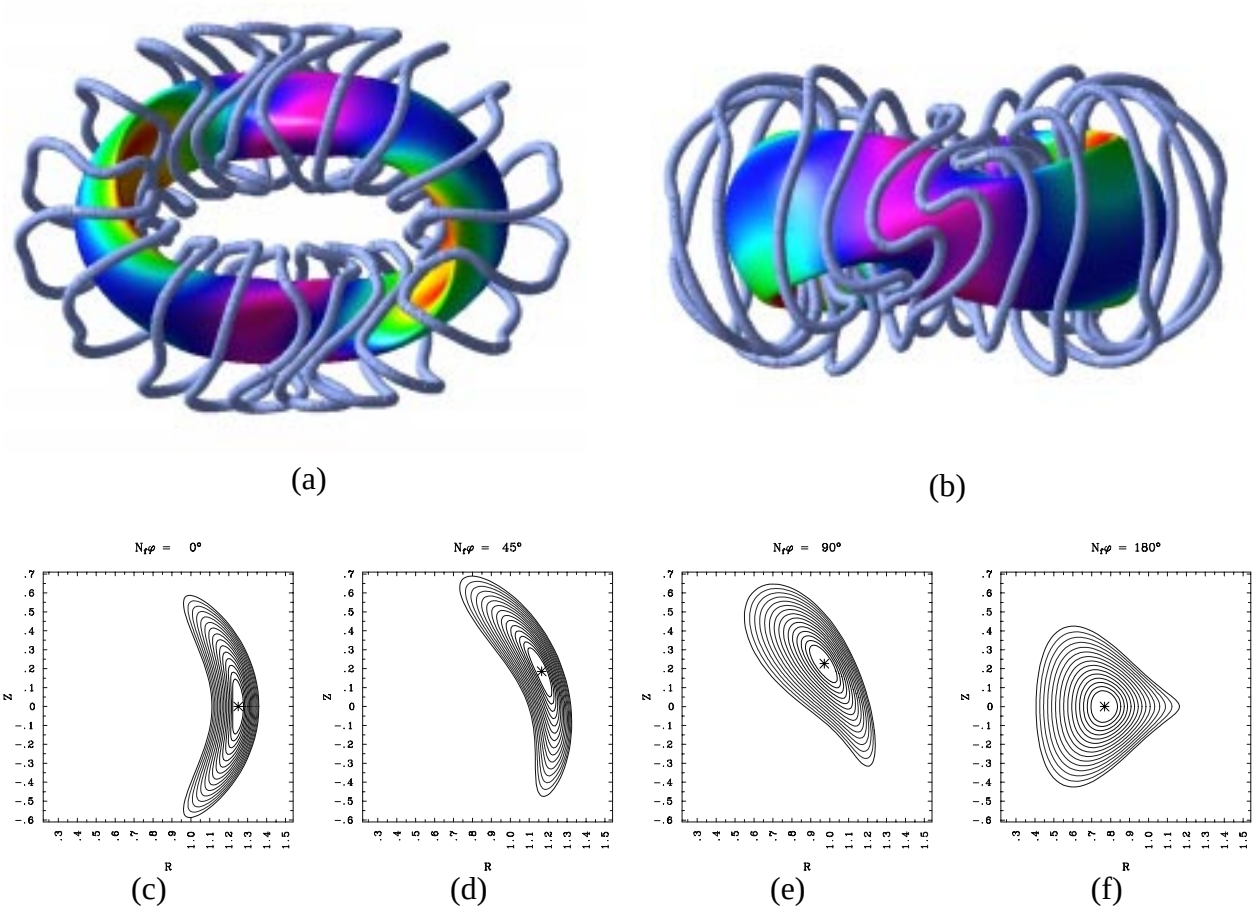

Fig. 2 – (a) Top and (b) side views of outer magnetic flux surface and coils with color contours showing magnetic field strength. Flux surface view plots at $N\zeta = 0°, 45°, 90°, 180°$

indicated as light blue filaments. The four coils near the top and bottom of Figure 2(a) are actually two split coils with each leg carrying half the current of the other coils.

IV. Coil Optimization

As indicated in Table 1, the physics optimization has already been weakly coupled to selected coil related figures of merit (e.g., complexity, curvature, and current density) which can be approximated by the NESCOIL[4] surface current model. These targets help guide the choice of plasma shapes into regions of parameter space where realizable coils will exist. However, the coils displayed in Figure 2(a) and (b) have not been generated from a continuous surface current distribution, but rather using the COILOPT[8] code. This approach solves for the optimal parameters in an explicit representation of modular coils on a toroidal winding surface that is well separated from the plasma boundary, together with the Fourier coefficients of the winding surface. A schmatic diagram of this process is shown in Figure 3. The primary target for this optimization is the reduction of the normal component of magnetic field on the outermost magnetic flux surface; in addition, a number of coil geometry and engineering penalty functions are also included. Examples are plasma-coil, coil-coil separation, coil current density and measures of coil curvature.

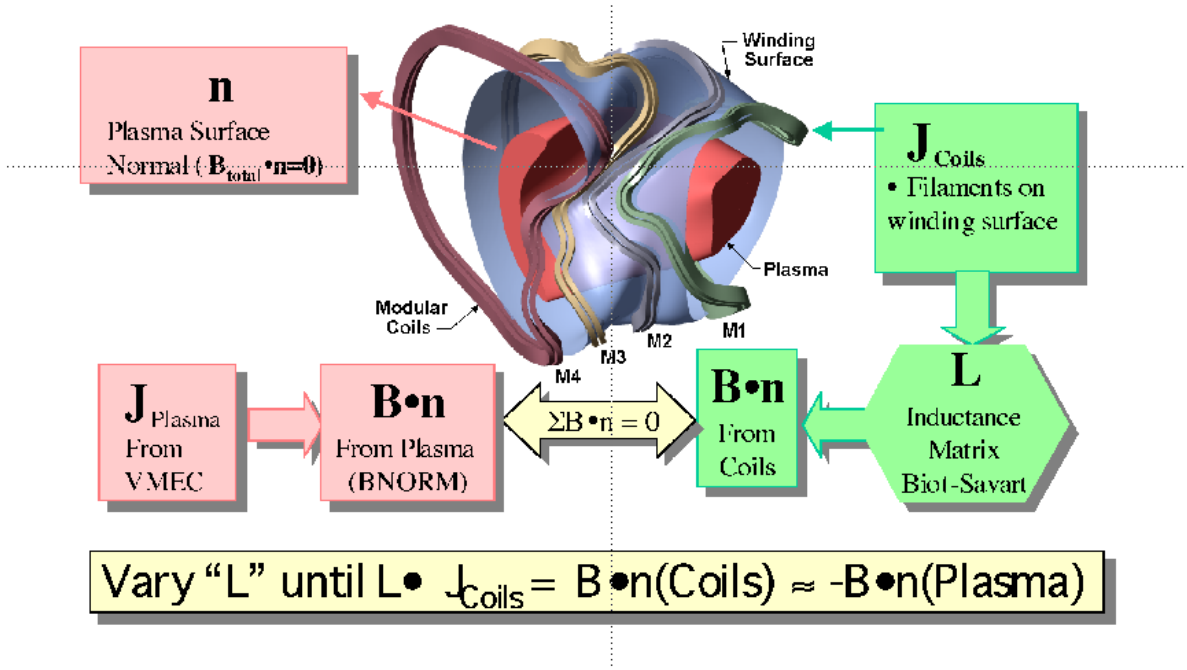

Fig. 3 – Schematic diagram of the COILOPT coil design process.

We have chosen a QPS coil optimization model based on 16 coils (4 coil types, where the winding packs of the pair of coils on the center of the long section follow follow independent paths) with no coils on the symmetry planes, uniform modular coil currents, 3 pairs of vertical field coils with fixed position and variable current, and 12 TF (toroidal field ∝ 1/R) coils. The latter field is in a direction opposite to the modular coil-produced TF. This increases the modular coil currents, but decreases the toroidal variation of the coils, thus improving the coil-coil separation. We are able to obtain good flux surface reconstruction and preserve the physics properties for QPS devices with average field errors on the outer surface [$\delta B_{avg} = (1/A)\oint\!\!\oint dA|\mathbf{B}\cdot\mathbf{n}|/|\mathbf{B}|$] of ~ 0.76%.

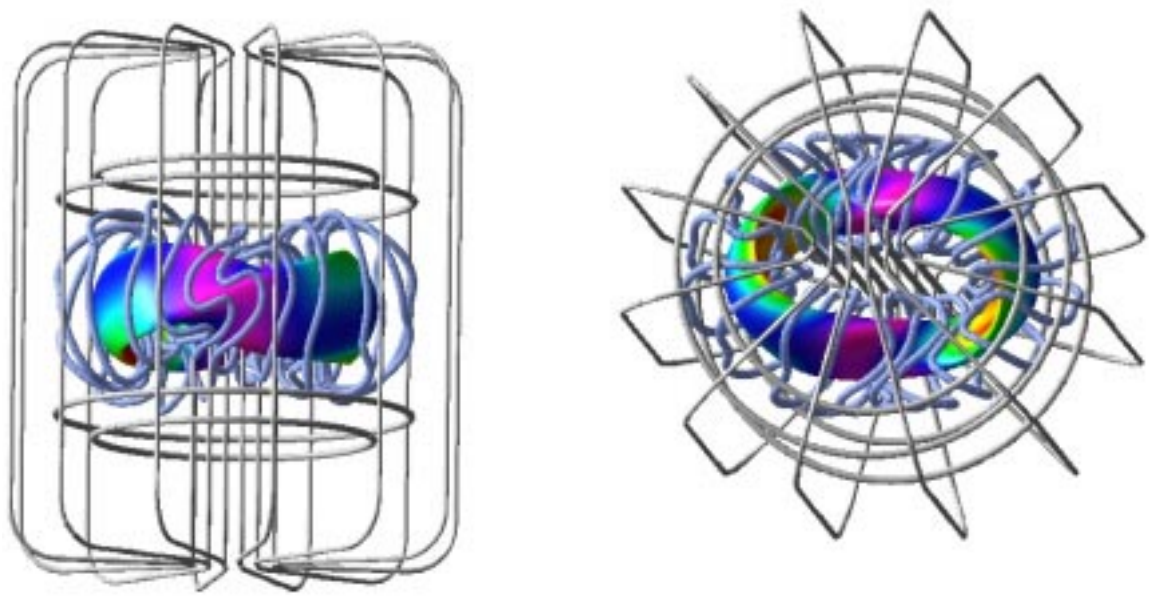

Fig. 4 – (a) Side and (b) top views of outer magnetic flux surface and the three coil sets (modular, vertical, toroidal) with color contours showing magnetic field strength.

Two views of the QPS outer magnetic surface along with the three coil sets (the modular coils, the vertical field coils and the toroidal field coils) are shown in Figures 4(a) and 4(b). These combined modular, TF and VF coils meet a variety of QPS specific plasma and device constraints, such as adequate space in the center of the device for Ohmic solenoid and TF coils (~ 20 cm), sufficiently large minimum radius of curvature (~ 6 cm) and adequate coil-plasma spacing (~ 14 cm).

The combination of toroidal and vertical field coils will offer significant flexibility to explore the effect of differing magnetic field structures on plasma properties. An example of such flexibility is shown in Figure 5, where the vertical field currents are varied, causing the plasma magnetic axis to either shift inward (negative currents) or outward (positive currents). As can be seen, inward shifts improve collisionless transport properties (here we have used the effective ripple coefficient $\varepsilon_{eff}^{3/2}$ from the NEO[5] code) and reduce the ballooning growth rates and size of the unstable region. Outward shifts increase the ballooning unstable region; this effect could prove to be a useful control for testing ballooning stability in an experiment. These variations were carried out at constant plasma pressure with <β> varying from about 1.9% in the shifted in case to 2.1% in the shifted out case; the toroidal field current was set at I_{tor} = –300 kAmps.

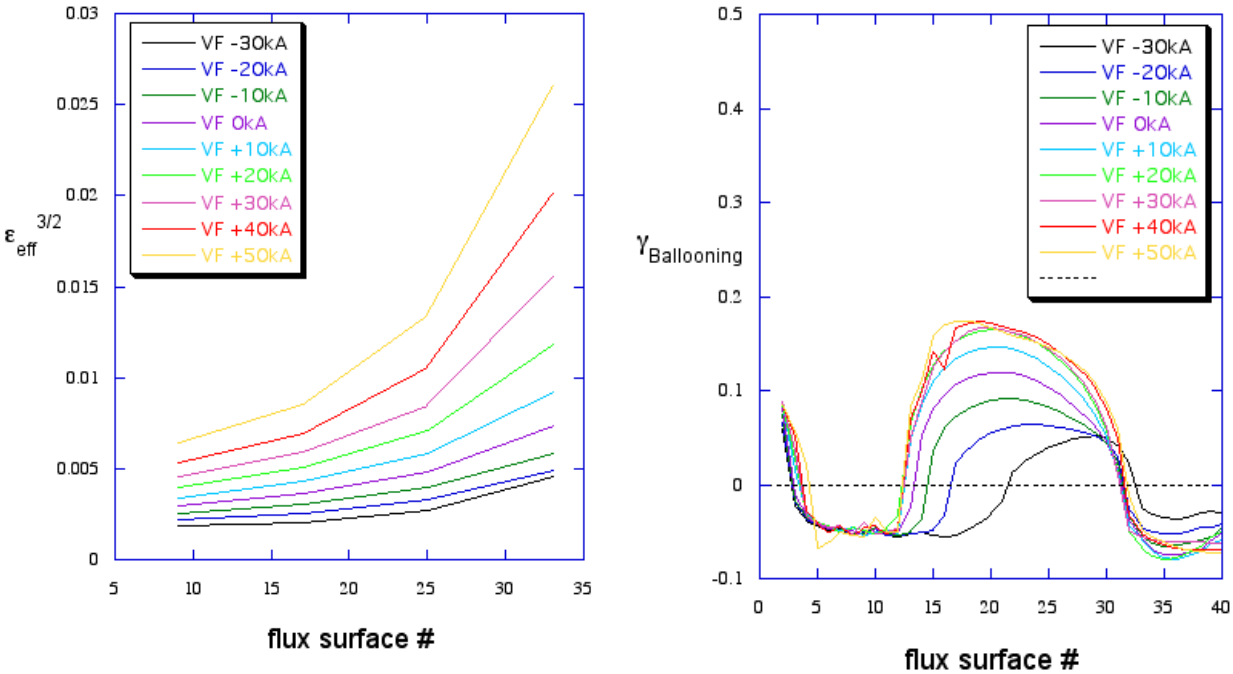

Fig. 5 – (a) Dependence of collisionless transport coefficient on vertical field coil current, (b) dependence of ballooning growth rate on vertical field coil current (here positive currents shift the magnetic axis outward and negative currents shift it inward).

V. Merged Plasma – Coil Optimization

Recently, the STELLOPT and COILOPT codes have been successfully merged together to form an optimization tool that will simultaneously target physics and coil engineering criteria in a more comprehensive manner. This approach is generally used only after the separate physics and coil optimizations have identified good candidate configurations for a design point. This merged optimizer has allowed the designer to identify nearby neighboring equilibria that have close to the same physics properties as the initial one, but with significantly better coils (i.e., lower field errors, easier to build). It has also, in some cases, been of use in identifying nearby

equilibria whose outer flux surface shapes have a smoother structure than those that are initially reconstructed from coils based only on the COILOPT calculation.

VI. Conclusions

A systematic plasma optimization and modular coil synthesis procedure has been developed and applied toward the development of low aspect ratio stellarators with quasi-poloidal symmetry. The plasma and coil optimization codes have been designed to take good advantage of currently available parallel computing architectures, but yet allow easy incorporation of new independently developed optimization targets. This effort has lead to the QPS device which has two field periods, A = 2.7, levels of neoclassical transport that are substantially suppressed from the stellarator ISS95 scaling, first stability limits around $<\beta>$ ~ 2% and second stability regimes up to $<\beta>$ ~ 15%. Using the COILOPT code, modular coils have been developed for this device that have good engineering feasibility, but yet preserve the flux surface integrity and physics characteristics of the original plasma optimization. A combination of modular, vertical and toroidal field coils has resulted in an experimental design that should be able to drive a range of Ohmic current levels while providing good flexibility for testing transport and stability physics.

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