Impact of Plasma Current Profile and External Coil Currents on Ideal MHD Stability in the Quasi-Poloidal Stellarator

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Abstract. This work examines the impact of both the plasma current profile and external coil currents on ideal MHD stability in the Quasi-Poloidal Stellarator (QPS). The proposed QPS device is a two-field period, low-aspect ratio (A = 2.7) stellarator which has been optimized for quasi-poloidal symmetry. The reference coil set for QPS contains 20 modular coils with 5 different coil shapes with each of the 4 coils of a given shape forming a group with a separate power supply. In addition to the modular coils, there are also a set of toroidal field coils and 3 pairs of vertical field coils. This coil set allows for flexible variation of the magnetic configuration. QPS has been designed to run with a bootstrap aligned current profile (a total current of ≤ 40 kA) at $\langle\beta\rangle$ ≤ 2% but will also have the capability of driving Ohmic current. By varying both the external coil currents and the plasma current profile, the ideal MHD stability properties of QPS can be greatly impacted. The infinite-n ballooning β limit can vary by a factor of ~2. The marginal ballooning β limit is lowest with an Ohmic current profile, highest with a bootstrap current profile, and can be varied between these limits with a combination of Ohmic and bootstrap current. For a given plasma current and pressure profile, variation of the modular and vertical field coil currents can change the β limit by a factor of ~1.5. As expected, finite-n ballooning modes (for n up to 19) have higher β limits, more than 200% higher than the infinite-n limit with a strong dependence on the plasma current. Kink and vertical modes are stable in QPS to much higher plasma β (> 5%).

I. Introduction

The proposed Quasi-Poloidal Stellarator (QPS) is a very-low-aspect ratio (A = 2.7) stellarator with a $|B|$ spectrum that exhibits quasi-poloidal symmetry. The QPS experiment will test the concept of quasi-poloidal symmetry as a pathway for obtaining adequate neoclassical confinement at very-low aspect ratio and in addition will explore the physics of MHD stability and anomalous transport in a quasi-poloidal symmetric device. A crucial aspect in the design of the QPS experiment has been the development of an optimized MHD equilibrium and its confinement and stability properties. Here, we present a detailed numerical analysis of the MHD stability properties of a reference QPS configuration, with a particular focus on the impact of external coil currents and plasma current on MHD stability. The primary modes of concern for stellarators are Mercier (global interchange) and ballooning modes (local interchange). For QPS, the nonzero plasma current makes kink or vertical mode instability a possibility. The numerical analysis of all of these modes in QPS is discussed below.

II. Equilibrium

The design of the QPS experiment incorporated a current profile fully compatible ("aligned") with the bootstrap current. The 3D equilibrium code VMEC [2] is used to obtain

the finite-β equilibria. The reference coil set and last closed flux surface of the reference configuration at $\langle\beta\rangle$ = 2% are shown in Fig. 1. The reference configuration has 300 kA of current in the modular coils, -129, -76, and 0 kA in the small, medium and large vertical field coils, and –25 kA in the toroidal field coils. The VMEC equilibrium current, the pressure profile, and the rotational transform profile for the reference configuration are shown in Fig. 2. The pressure profile used to test stability is a simple quadratic profile, $p = p_0(1 - S)^2$, where S is the normalized toroidal flux [$S \sim (r/a)^2$]. The reference configuration has 11 kA of net parallel bootstrap current at $\langle\beta\rangle$ = 2%. This value of bootstrap current is 1/4 the full collisionless bootstrap current and is appropriate for the more collisional operating regime expected in QPS. In the following, when testing stability at higher β, the bootstrap current is scaled proportionately with $\langle\beta\rangle$(e.g., 22 kA of current at $\langle\beta\rangle$= 4%).

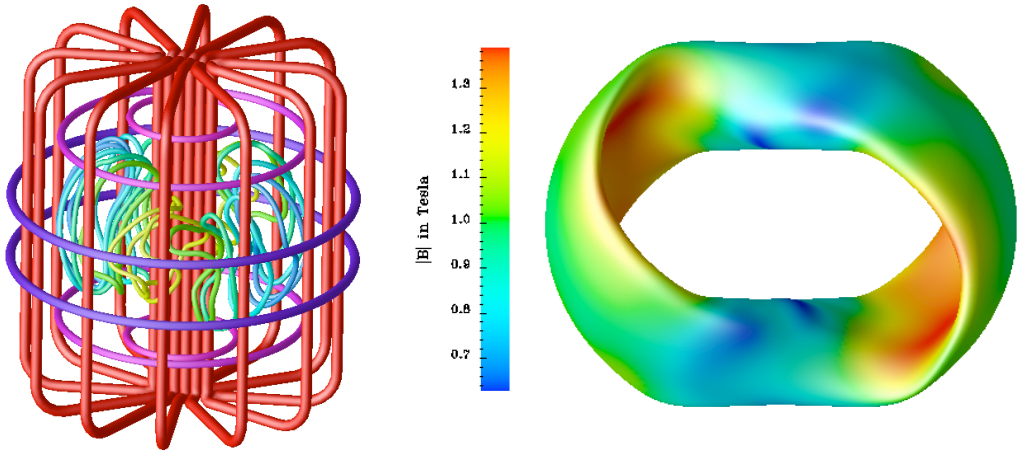

Figure 1. (a) Modular, vertical field, and toroidal field coils for the reference QPS configuration and (b) last-closed flux surface of a QPS plasma with $\langle\beta\rangle$ = 2.0%, I_{plasma} = 11 kA, and coil currents of I_{mod} = 300 kA, I_{VF1} = –129 kA, I_{VF2} = –76 kA, I_{VF3} = 0 kA, and I_{TF} = –25 KA.

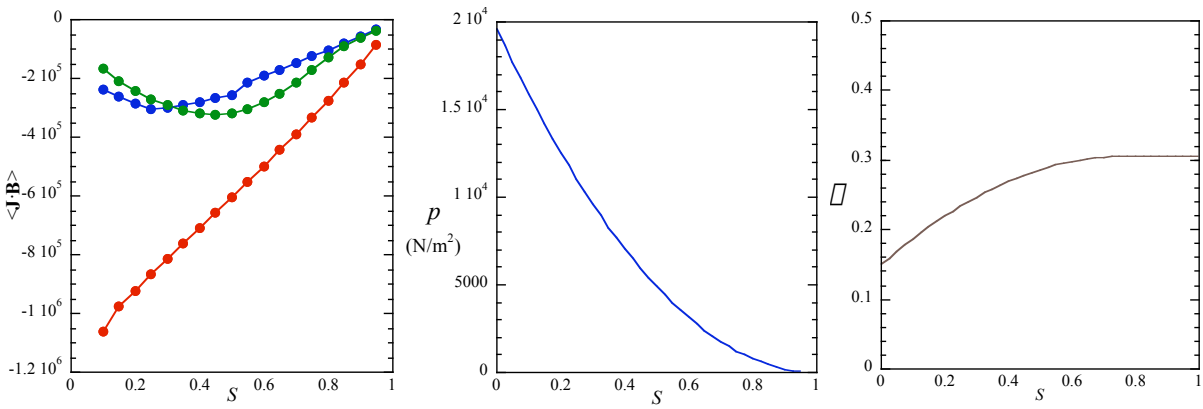

Figure 2. Profiles of (a) the field-aligned current, (b) the pressure profile, and (c) the rotational transform profile for the reference QPS configuration.

III. Mercier Stability

The QPS reference configuration is stable to Mercier (global interchange) modes over the entire plasma cross-section except for a few isolated resonances as shown in Fig. 3a. This configuration has a substantial well over the entire plasma cross section due to a combination of helical curvature and a small Shafranov shift (see Fig. 3b). It is stabilized primarily by the well rather than the shear (bending) terms in the Mercier criterion. This applies everywhere except near the edge, where the destabilizing geodesic curvature is small anyway. The terms

arising from the net parallel bootstrap current are stabilizing but small. The strong well stabilization implies that high-n resistive modes should also be stable in this configuration.

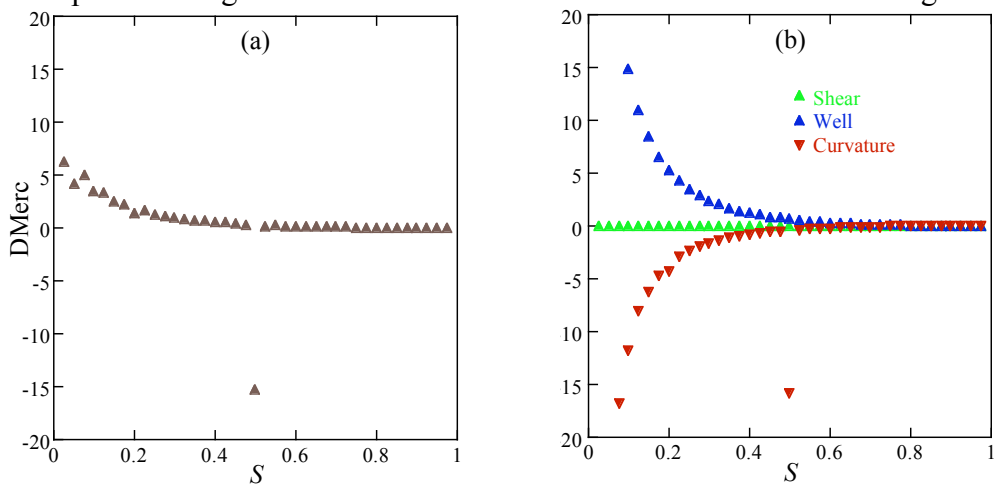

Figure 3. (a) Mercier stability criteria for the QPS reference configuration and (b) the well (green diamonds) and shear (blue diamonds) stabilizing contributions to the criteria.

IV. Ballooning Mode Stability

The QPS reference configuration is sensitive to ballooning modes as analyzed by the Code for Ballooning Rapid Analysis (COBRA) [3]. These set the β MHD stability limits for the reference configuration. The field line-bending term is weakened as a result of the small number of field periods (the connection length R/N is relatively long). This seems to be the dominant adverse effect on stability in QPS.

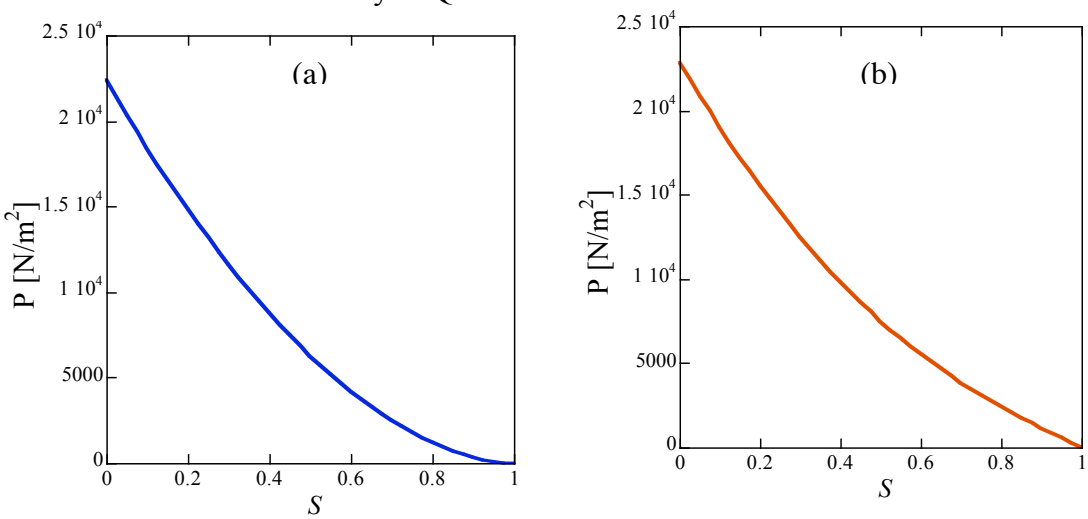

Figure 4. (a) Optimized pressure profiles for the QPS reference configuration, (a) with $dp/dS = 0$ at the edge and (b) with $dp/dS \neq 0$ at the edge.

The pressure profile for the reference configuration has been optimized for ballooning stability at $\langle\beta\rangle = 2\%$, both with the constraint $dp/dS = 0$ at the plasma edge to avoid edge currents and letting $dp/dS \neq 0$ at the plasma edge to allow for an edge pedestal. The optimized pressure profiles are shown in Figs. 4a and 4b. The ballooning growth rates (positive is unstable) of the reference configuration with these profiles is shown in Fig. 5a and 5b, respectively. For the pressure profile with zero edge gradient, as β is increased above 2% (a region of instability appears in the center of the plasma for $\langle\beta\rangle > 2.1\%$ (Fig. 5a). For the equilibria with an edge pressure gradient, the β limits are higher (Fig. 5b). Those

configurations had a β limit of β~ 2.4%. This class of profiles may develop naturally as a result of H-mode physics.

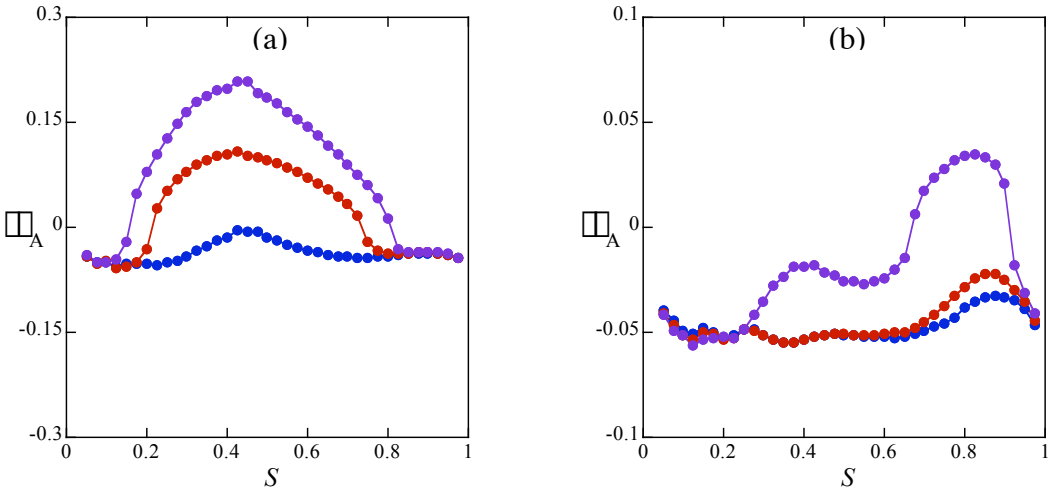

Figure 5. Ballooning growth rate normalized to the Alfvén time from COBRAVMEC for β = 2.0 (blue), 2.25 (red), and 2.5% (purple) with (a) dp/dS = 0 and (b) dp/d$S \neq 0$ at the plasma edge.

In order to be able to test ballooning stability at low β in the QPS experiment, we have also used the stellarator optimization code to search for configurations with *degraded* ballooning stability. The external coil currents and plasma profiles were varied to produce a plasma with the reference QPS coil set that was ballooning unstable at β~ 1.5%. For this case, the plasma current profile was a combination of bootstrap and Ohmic current (see Fig. 6a) for a total current of 41 kA. The external currents were 390, 390, 250, 230, and 395 kA in the modular coils, 0, 32, and –426 kA in the vertical field coils, and –21 kA in the toroidal field coils. The field-aligned current profile, pressure profile, and ballooning growth rate for this case are shown in blue in Fig. 6. If the external currents are allowed to vary to more extreme values, lower ballooning b limits are possible. The profiles and growth rates for such a case with β= 1.2% is shown in red in Fig. 6. For this case, the modular coil currents are 532, 643, 111, 77, and 395 kA in the modular coils, 0, 32, and –426 kA in the vertical field coils, and –21 kA in the toroidal field coils.

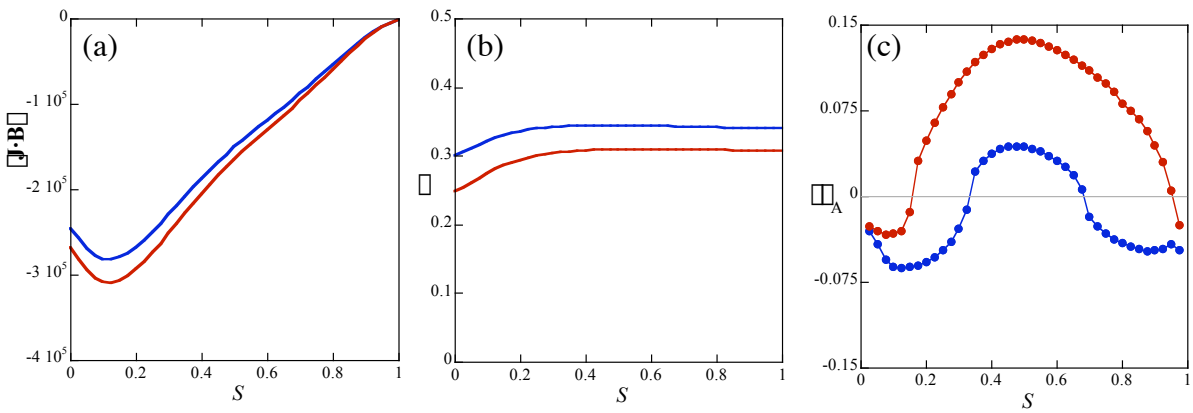

Figure 6. Profiles of (a) the field-aligned current, (b) the rotational transform profile, and (c) the ballooning growth rate from COBRAVMEC for the reference QPS configuration with modified external coil currents at β= 1.6% (blue) and with extreme external coil currents at β= 1.2% (blue).

While infinite-n ballooning stability is useful to set a lower bound on the ideal MHD β limit, there are stabilizing terms such as FLR effects and two-fluid effects which will stabilize the very short wavelength instabilities. In order to obtain a better indication of the expected

ballooning β limit in QPS the TERPSICHORE stability code is used to analyze finite-n (up to n = 19) ballooning mode stability. The choice of a maximum n of 19 is made primarily for computational efficiency. To accurately estimate the finite-n ballooning stability, a high-resolution VMEC equilibrium is required with 201 radial surfaces and 162 Fourier modes used in the equilibrium calculation. This resolution is higher than is used in the equilibrium optimization (where 41 radial surfaces and 113 Fourier modes are used).

To test finite-n ballooning stability, the simple quadratic pressure profile is used with a series of plasma equilibria at $\langle\beta\rangle$ = 2, 3, 3.25, 3.5, 4, and 4.5% with toroidal plasma current scaled as 11, 16, 17.5, 19, 21, and 24 kA, respectively. These correspond to roughly 1/4 the collisionless bootstrap current predictions. The normalized finite-n ballooning eigenvalues from TERPSICHORE are shown in Fig. 4.6 with negative values indicating instability (eigenvalue ~ β^2). This QPS configuration is stable to finite-n ballooning modes through $\langle\beta\rangle$ = 4.5%. In order to test the effect of plasma current on ballooning stability, a second series of equilibria with $\langle\beta\rangle$ = 2, 3, 3.25, 3.5, and 4% with 21, 32, 34, 37, and 42 kA, respectively. These correspond to roughly 1/2 the full collisionless bootstrap current. With these current values, this QPS configuration is stable to finite-n ballooning modes up to $\langle\beta\rangle$ = 3% and is unstable for $\langle\beta\rangle$ > 3% (Fig. 4.6).

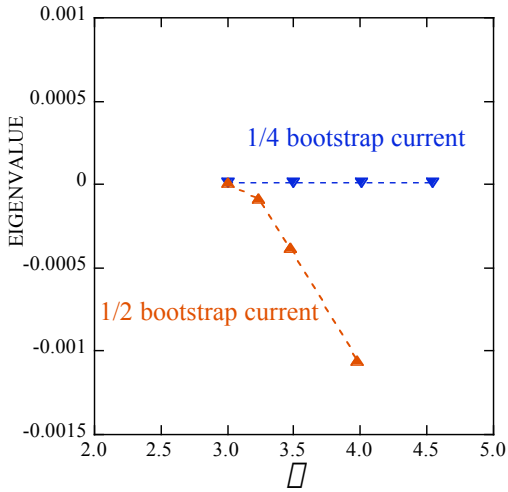

Figure 7. Finite-n ballooning mode eigenvalues from TERPSICHORE as a function of β with both the base (blue) and twice the base (red) bootstrap current.

IV. Kink and Vertical Mode Stability

The finite current in the QPS configuration makes it necessary to address the issue of low-n ideal MHD modes which can be destabilized by current and pressure gradients. The TERPSICHORE stability code was used to analyze vertical (n = 0) and kink (n = 1) mode stability of the reference configuration. Note that these two modes include other low-n values that couple together. This configuration is stable to both vertical and kink modes at $\langle\beta\rangle$ = 2%. To test the stability of QPS to kink and vertical modes, a series of equilibria with $\langle\beta\rangle$ = 3, 4, 4.5, 5, and 5.5%, with toroidal plasma current scaled as 16, 21, 23, 26, and 29 kA, respectively (again corresponding to roughly 1/4 the full collisionless bootstrap current). The kink and vertical mode eigenvalues from TERPSICHORE are shown in Fig. 4.7. This QPS configuration is stable to kink and vertical modes up to $\langle\beta\rangle$ = 5%. Only at the highest β tested, namely $\langle\beta\rangle$ = 5.5%, is this QPS configuration unstable to both a kink mode and a vertical mode (Fig. 4.7).

The vertical stability is due primarily to the external rotational transform. An analytic stability criterion for the vertical mode in a large aspect ratio current-carrying stellarator has been developed (for constant current density and constant external rotational transform) [5]. The stability criteria relates the fraction of edge rotational transform, $F = \iota_{external}/\iota_{total}$, to the axisymmetric elongation, κ: $F \geq (\kappa^2 - \kappa)/(\kappa^2 + 1)$. The external transform is stabilizing because the external poloidal flux enhances the field line bending energy relative to the current-driven term for the vertical instability. For the reference configuration, we have F = 0.697, κ = 2.765, and $(\kappa^2 - \kappa)/(\kappa^2 + 1) = 0.565$, resulting in the condition being well-satisfied. This criterion is met for all of the QPS reference plasmas and is consistent with stability to vertical modes for all of these.

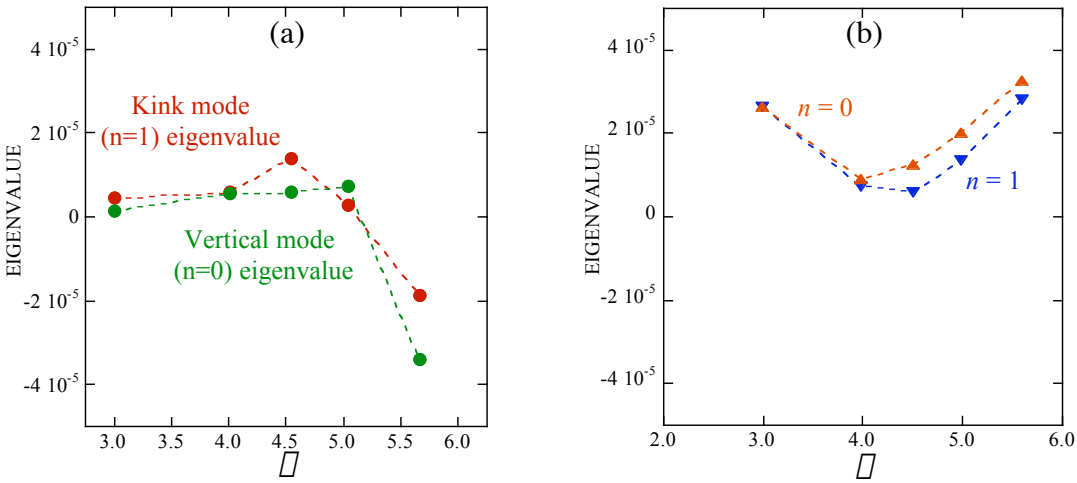

Figure 8. Kink (n = 1) and vertical (n = 0) mode eigenvalues from TERPSICHORE as a function of κ with both (a) the base and (b) twice the base bootstrap current.

IV. Conclusions

The proposed QPS experiment is a very-low apsect ratio stellarator with moderate, bootstrap-aligned plasma current. The MHD stability limits are set by ballooning modes rather than kink or vertical modes. The ballooning beta limit is approximately ~2%. The experimental ballooning beta limit may be higher than this if pressure profiles with edge pedestals are obtained. QPS is stable to Mercier modes up to β ~ 2.5%. Kink and vertical modes are not an issue for the QPS configuration due to the low plasma current (~40 kA of neoclassical bootstrap current at β = 2%). QPS plasmas are stable to kink and vertical modes up to β = 2%.

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