Neoclassical Parallel Momentum Balance and Flow Damping in Quasi-Symmetric Stellarators

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2004 International Sherwood Theory Conference

Missoula, Montana, April 26-28, 2004

Moments Method for Stellarator Transport
(i.e., extension of NCLASS approach to 3D systems)

- QPS/NCSX/HSX have been strongly optimized:
  - so that neoclassical losses << anomalous losses
  - The remaining transport-related differences will be in the parallel momentum transport properties
- Recently, a theoretical framework has been developed that allows quantitative, self-consistent assessment of the parallel and perpendicular transport in 3D systems:
  - H. Sugama and S. Nishimura, Physics of Plasmas, **9 (**November, 2002) 4637.
  - Extends DKES transport coefficients (based on pitch-angle scattering operator) to include momentum-conservation and ion-electron frictional coupling effects.
  - Provides particle/energy fluxes, viscosity tensor, flows, and bootstrap current
- Calculation of flow velocity profiles for stellarators is motivated by:
  - Relevance to turbulence suppression/enhanced confinement regimes
  - Comparison with impurity line measurements
  - Impurity accumulation/shielding studies
- More accurate collisional bootstrap current prediction, and ambipolar electric field estimation

Moments Method Equations

Parallel momemtum balance relations

$$\left\langle \vec{B}\cdot\left(\nabla\cdot\vec{\Pi}_a\right)\right\rangle - n_a e_a \langle B E_\parallel \rangle = \langle B F_{\parallel a1} \rangle$$

$$\left\langle \vec{B}\cdot\left(\nabla\cdot\vec{\Theta}_a\right)\right\rangle = \langle B F_{\parallel a2} \rangle$$

Friction-flow relations

[S. Hirshman, D. J. Sigmar, Nuclear Fusion **21**, 1079 (1981)]

$$\begin{bmatrix} \langle BF_{\parallel a1}\rangle \\ \langle BF_{\parallel a2}\rangle \end{bmatrix} = \sum_b \begin{bmatrix} l_{11}^{ab} & -l_{12}^{ab} \\ -l_{12}^{ab} & l_{22}^{ab} \end{bmatrix} \begin{bmatrix} \langle Bu_{\parallel b}\rangle \\ \frac{2}{5p_b}\langle Bq_{\parallel b}\rangle \end{bmatrix}$$

• The parallel components of the viscous stress tensor are given in terms of the DKES eqn. (35) of H. Sugama, S. Nishimura, Phys. Plasmas **9**, 4637 (2002) :

$$\begin{bmatrix} \left\langle \vec{B}\cdot\left(\nabla\cdot\vec{\Pi}_a\right)\right\rangle \\ \left\langle \vec{B}\cdot\left(\nabla\cdot\vec{\Theta}_a\right)\right\rangle \\ \Gamma_a \\ Q_a/T_a \end{bmatrix} = \begin{bmatrix} M_{a1} & M_{a2} & N_{a1} & N_{a2} \\ M_{a2} & M_{a3} & N_{a2} & N_{a3} \\ N_{a1} & N_{a2} & L_{a1} & L_{a2} \\ N_{a2} & N_{a3} & L_{a2} & L_{a3} \end{bmatrix} \begin{bmatrix} \langle u_{\parallel a}B\rangle/\langle B^2\rangle \\ \frac{2}{5p_a}\langle q_{\parallel a}B\rangle/\langle B^2\rangle \\ X_{a1} \\ X_{a2} \end{bmatrix}$$

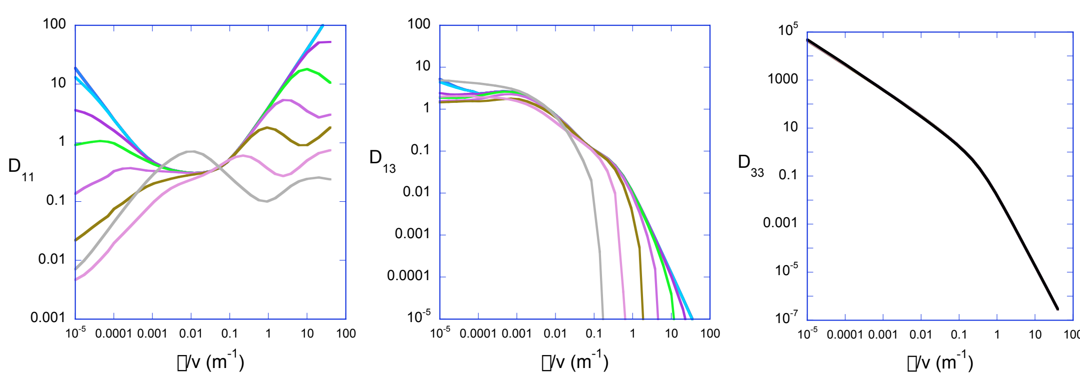
where $\left[M_{aj}, N_{aj}, L_{aj}\right] = n_a \frac{2}{\sqrt{\pi}} \int_0^\infty dK \sqrt{K} e^{-K} \left(K - \frac{5}{2}\right)^{j-1} \left[M_a(K), N_a(K), L_a(K)\right]$

$$M_a(K) = \frac{m_a^2}{T_a}\left[\nu_D^a(K)\right]^2 D_{33}(K)\left[1 - \frac{3m_a\nu_D^a(K)D_{33}(K)}{2T_aK\langle B^2\rangle}\right]^{-1}$$

$$N_a(K) = \frac{m_a}{T_a}\nu_D^a(K)D_{13}(K)\left[1 - \frac{3m_a\nu_D^a(K)D_{33}(K)}{2T_aK\langle B^2\rangle}\right]^{-1}$$

$$L_a(K) = \frac{1}{T_a}\left[D_{11}(K) - \frac{B^2 v^2 \nu_D^a}{3\Omega_a^2}\langle \tilde{U}^2\rangle\right]$$

$$+ \frac{3m_a\nu_D^a(K)\left[D_{13}(K)\right]^2}{2T_a^2K\langle B^2\rangle}\left[1 - \frac{3m_a\nu_D^a(K)D_{33}(K)}{2T_aK\langle B^2\rangle}\right]^{-1}$$



Radial fluxes, bootstrap current, and parallel, poloidal, toroidal flow velocities are obtained via the parallel force balance relation:

Radial particle flows required for ambipolar condition ⇒ self-consistent energy fluxes and bootstrap currents

$$\begin{bmatrix} \Gamma_e \\ q_e/T_e \\ \Gamma_i \\ q_i/T_i \\ J_{BS}^E \end{bmatrix} = \begin{bmatrix} L_{11}^{ee} & L_{12}^{ee} & L_{11}^{ei} & L_{12}^{ei} & L_{1E}^{e} \\ L_{21}^{ee} & L_{22}^{ee} & L_{21}^{ei} & L_{22}^{ei} & L_{2E}^{e} \\ L_{11}^{ie} & L_{12}^{ie} & L_{11}^{ii} & L_{12}^{ii} & L_{1E}^{i} \\ L_{21}^{ie} & L_{22}^{ie} & L_{21}^{ii} & L_{22}^{ii} & L_{2E}^{i} \\ L_{E1}^{e} & L_{E2}^{e} & L_{E1}^{i} & L_{E2}^{i} & L_{EE} \end{bmatrix} \begin{bmatrix} X_{e1} \\ X_{e2} \\ X_{i1} \\ X_{i2} \\ X_E \end{bmatrix}$$

where $X_{a1} \equiv -\frac{1}{n_a}\frac{\partial p_a}{\partial s} - e_a \frac{\partial \Phi}{\partial s}, \quad X_{a2} \equiv -\frac{\partial T_a}{\partial s}, \quad X_E = \langle B E_\parallel \rangle / \langle B^2 \rangle^{1/2}$

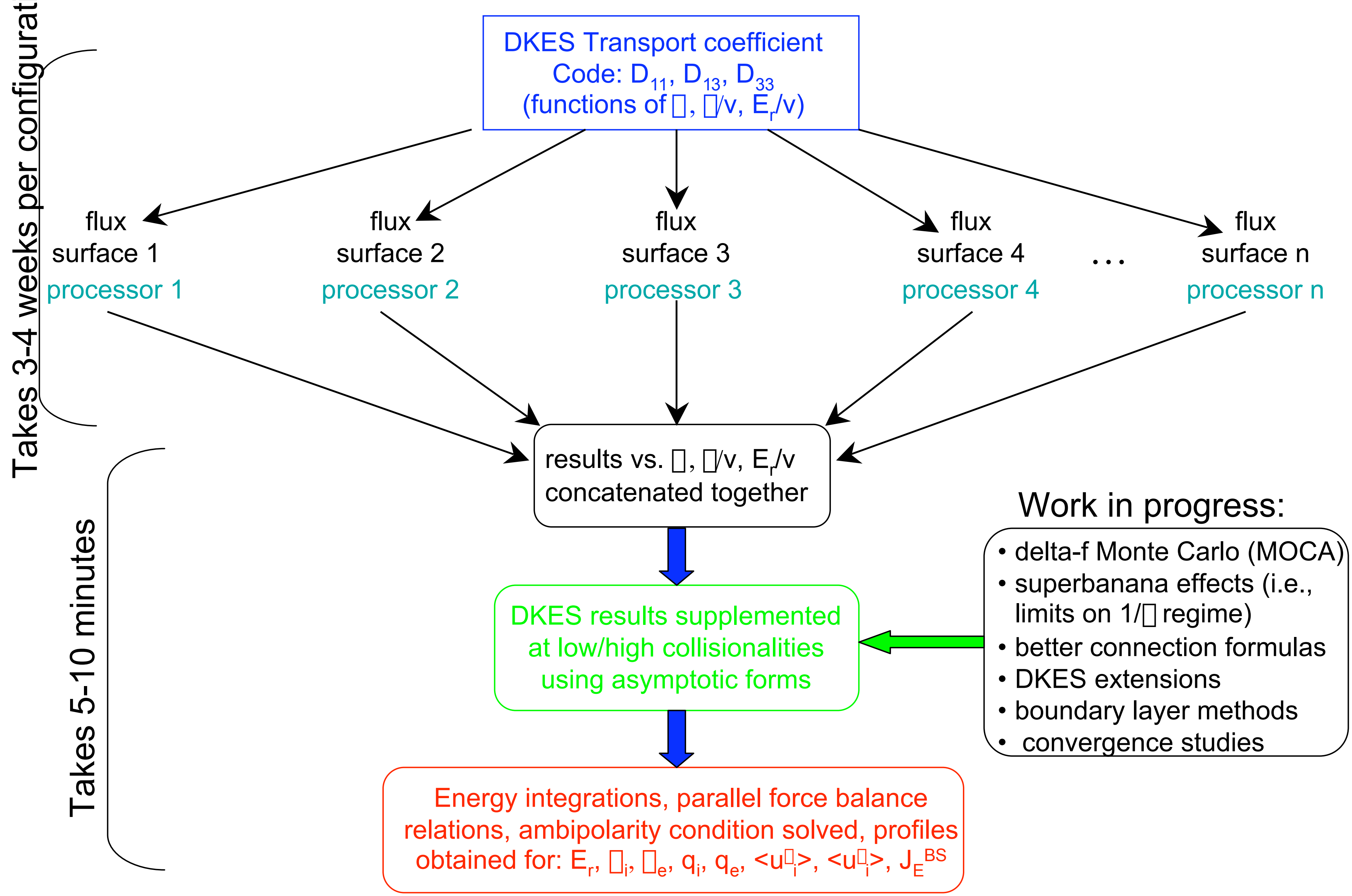
Parallel mass and energy flows:

$$\begin{bmatrix} \langle B u_{\parallel i} \rangle \\ \frac{2}{5 p_i} \langle B q_{\parallel i} \rangle \end{bmatrix} = \left(\frac{1}{\langle B^2 \rangle} \begin{bmatrix} M_{i1} & M_{i2} \\ M_{i2} & M_{i3} \end{bmatrix} + \begin{bmatrix} l_{11}^{ii} & -l_{12}^{ii} \\ -l_{12}^{ii} & l_{22}^{ii} \end{bmatrix} \right)^{-1} \begin{bmatrix} N_{i1} & N_{i2} \\ N_{i2} & N_{i3} \end{bmatrix} \begin{bmatrix} X_{i1} \\ X_{i2} \end{bmatrix}$$

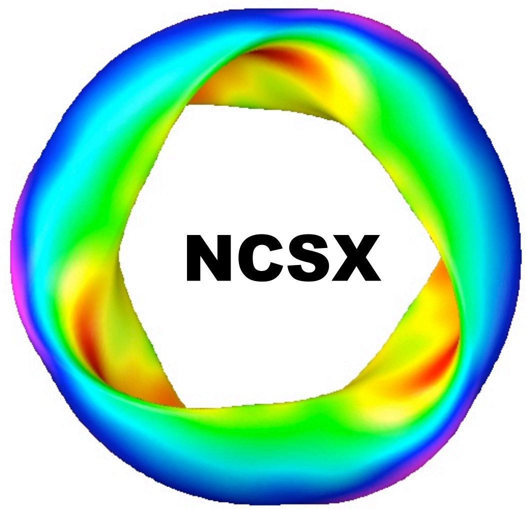
Poloidal and toroidal (contravariant) flow velocities:

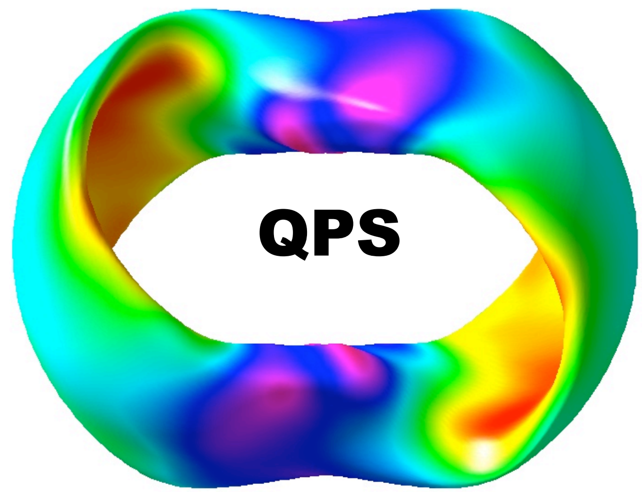
$$\begin{bmatrix} \langle \mathrm{u}_{\mathrm{i}}^{\theta} \rangle / \chi' \\ \langle \mathrm{u}_{\mathrm{i}}^{\zeta} \rangle / \psi' \end{bmatrix} = \frac{4\pi^2}{V'} \begin{bmatrix} 1 & -B_\zeta / \left(\chi' \langle B^2 \rangle \right) \\ 1 & B_\theta / \left(\chi' \langle B^2 \rangle \right) \end{bmatrix} \begin{bmatrix} \langle B u_{\parallel i} \rangle / \langle B^2 \rangle \\ X_{i1} \end{bmatrix}$$

Parallel Environment for Neoclassical Transport Analysis (PENTA)



- Quasi-symmetric stellarators based on the three forms of quasi-symmetry are now either operational or planned within the U.S. fusion program
  - HSX: quasi-helical symmetry |B| ~ |B|(m□□n□)
  - NCSX: quasi-toroidal symmetry |B| ~ |B|(□)
  - QPS: quasi-poloidal symmetry |B| ~ |B|(□)





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Princeton Plasma Physics Lab.

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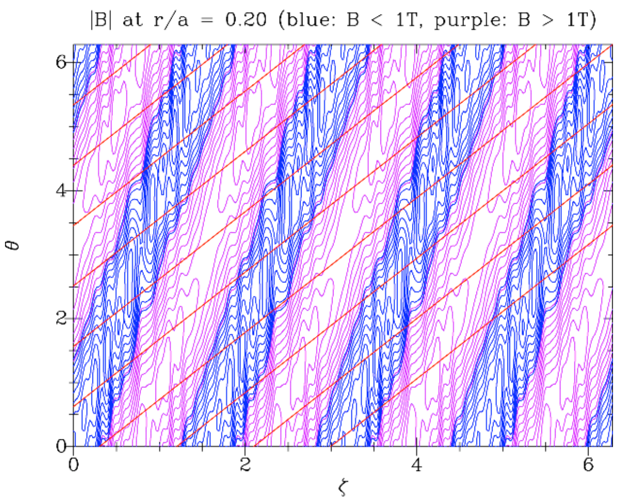
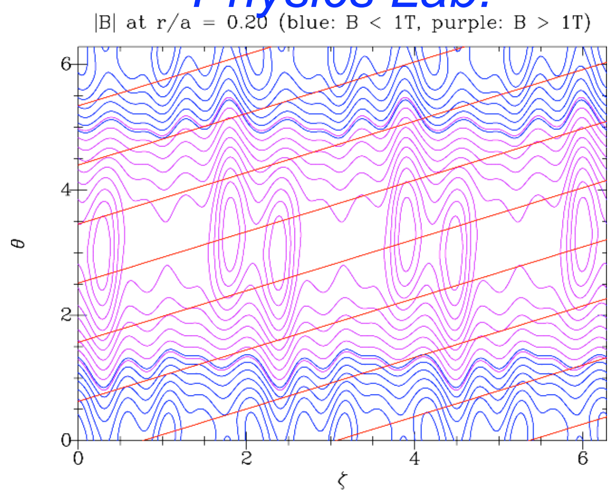
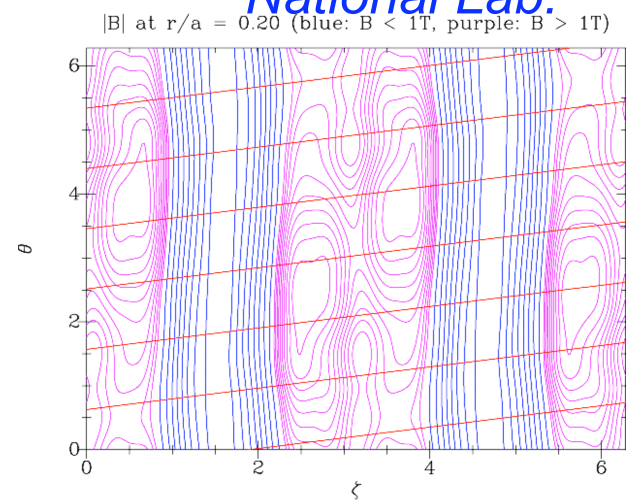
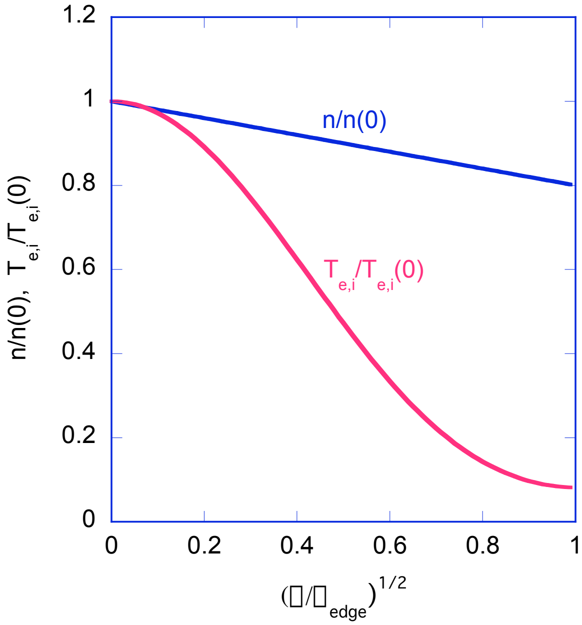
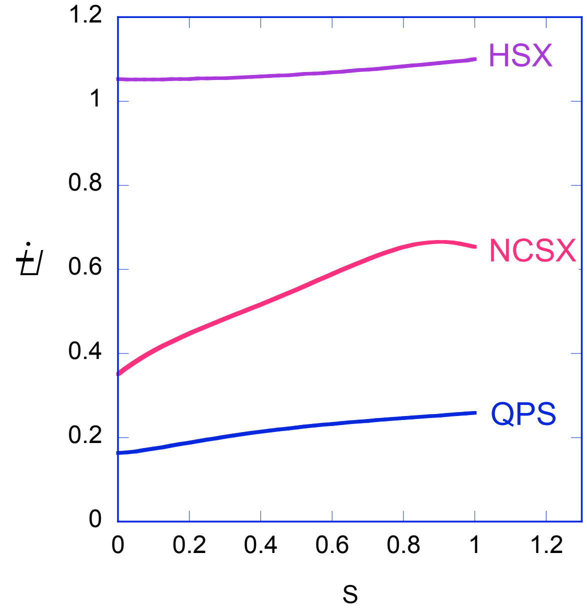






Profiles and parameters





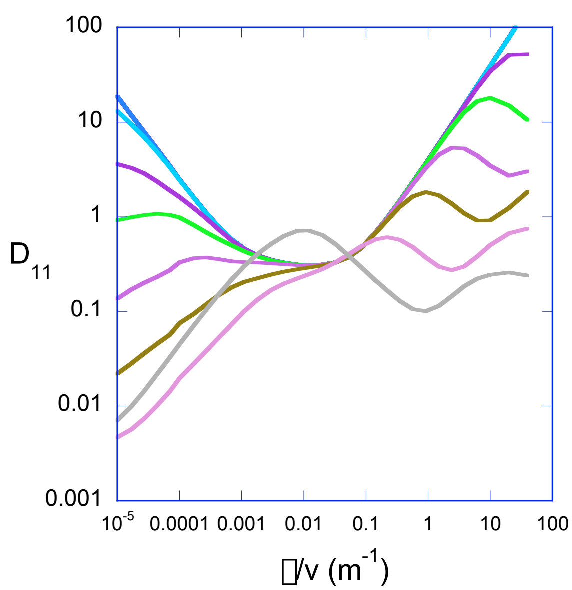
ECH Regime:

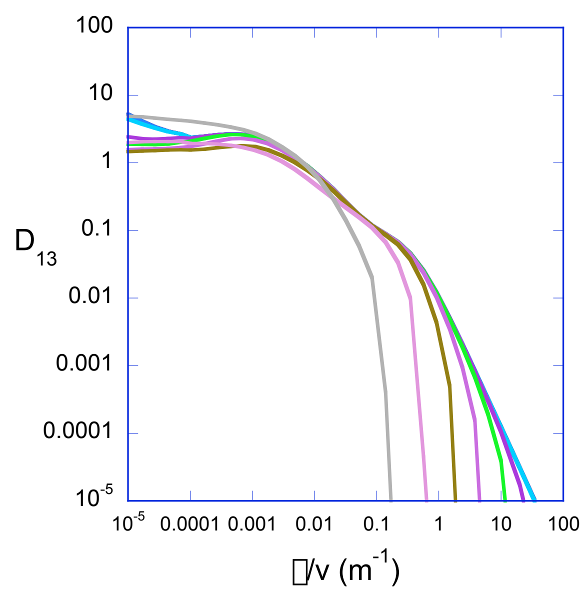
$n(0) = 2 \times 10^{19}\ m^{-3}$
$T_e(0) = 2.1$ keV
$T_i(0) = 0.2$ keV

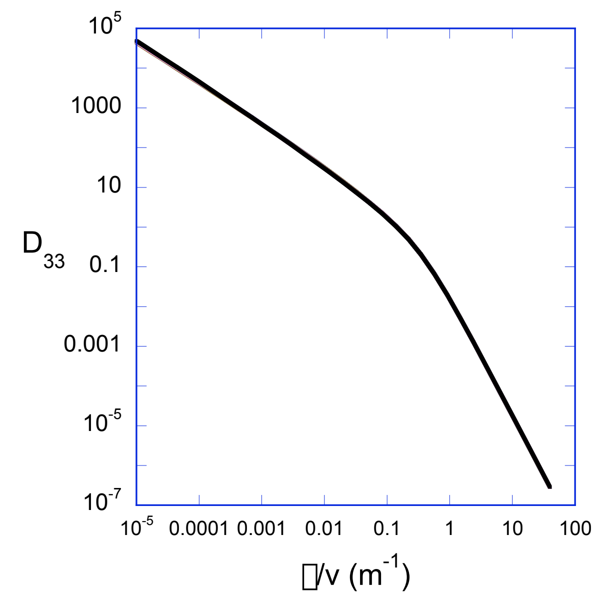
ICH Regime:

$n(0) = 8.3 \times 10^{19}\ m^{-3}$
$T_e(0) = 0.53$ keV
$T_i(0) = 0.38$ keV

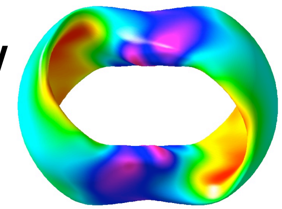
Typical DKES monoenergetic transport coefficients



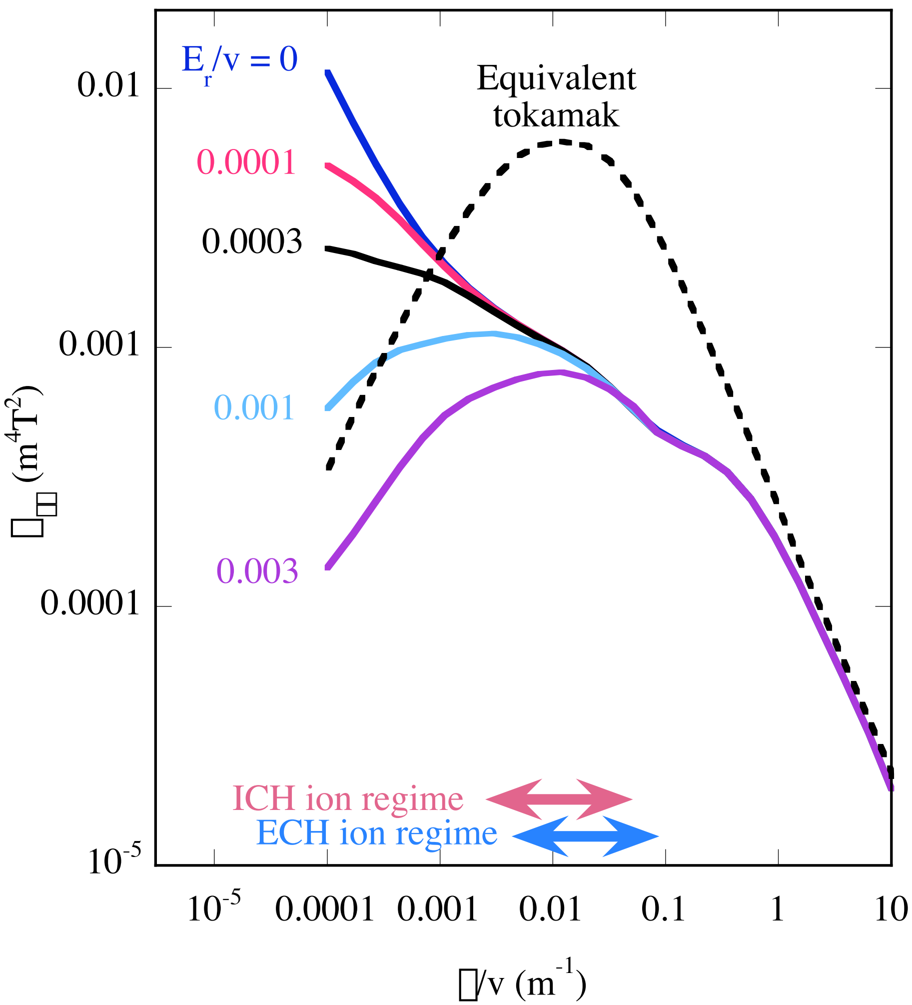




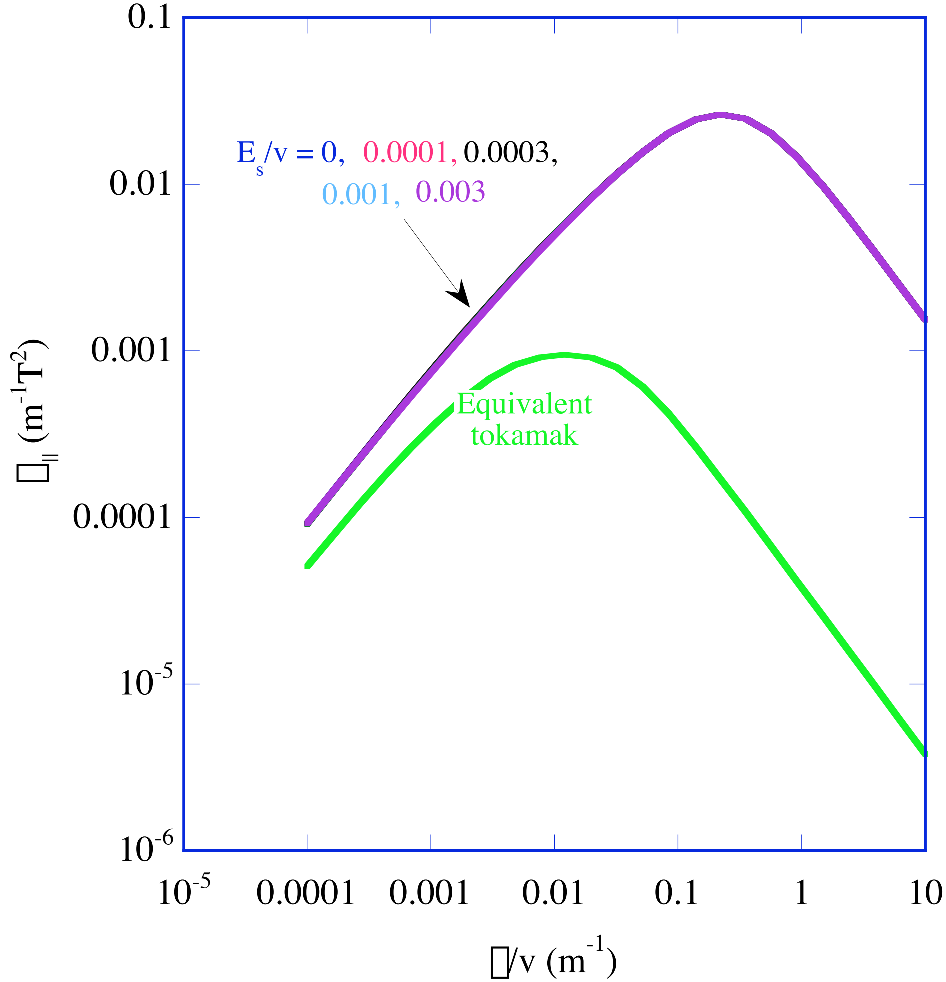
QPS - viscosities show strongly reduced poloidal flow damping from an equivalent axisymmetric device



poloidal - 10 x less than tokamak

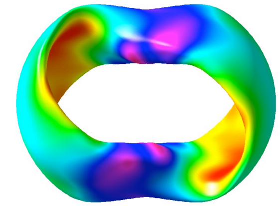


parallel



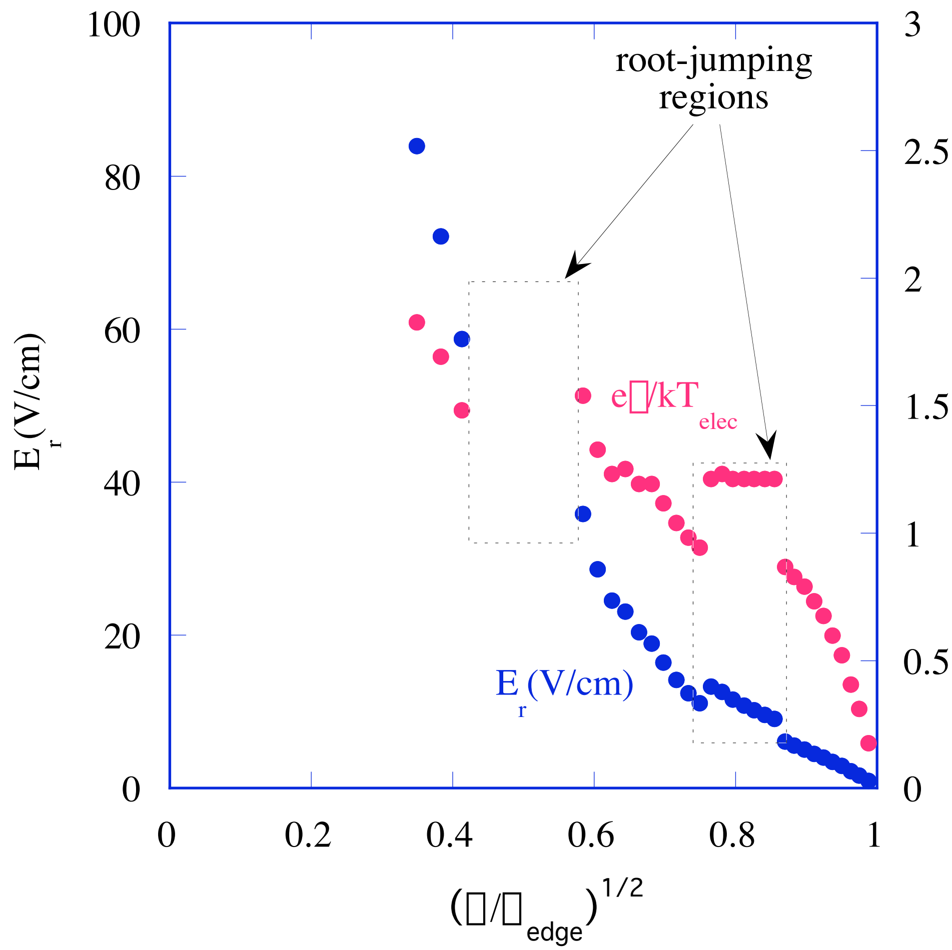
QPS - ECH regime

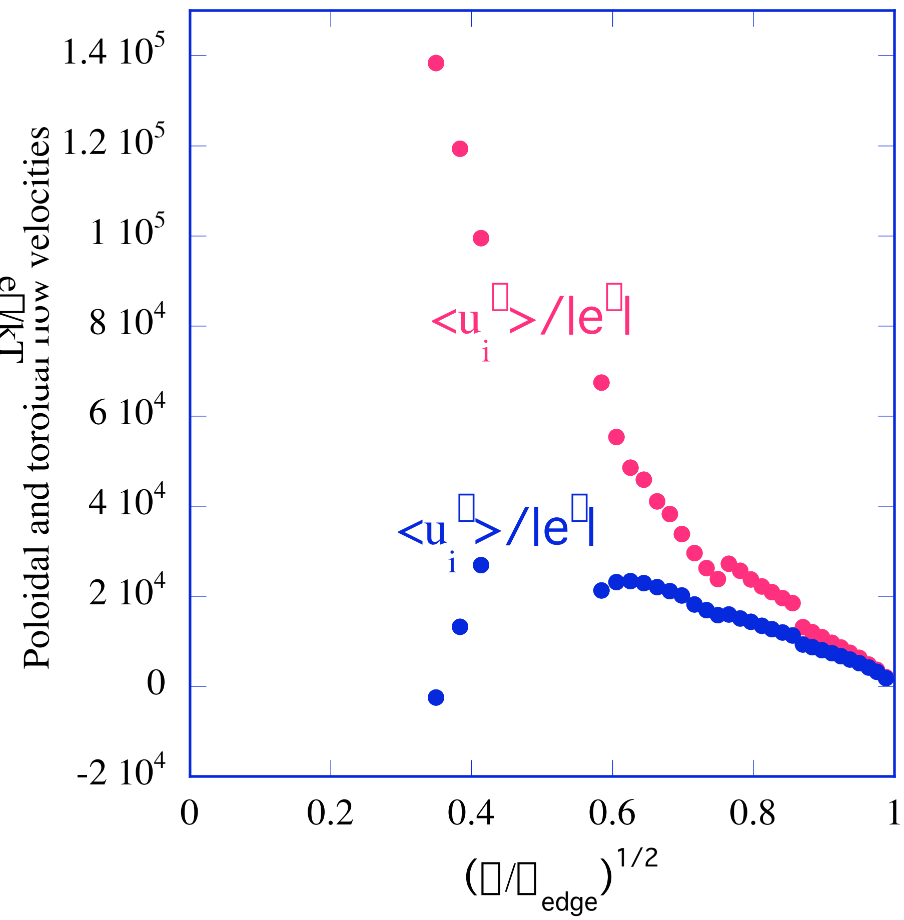
electric field and flow velocities



Toroidal flow suppressed - poloidal flow dominates

n(0) = 2 x 10^{19} m^{-3}, $T_e(0)$ = 2.1 keV, $T_i(0)$ = 0.2 keV





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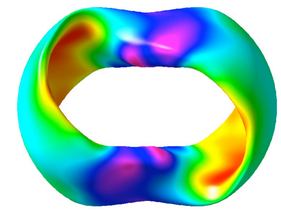

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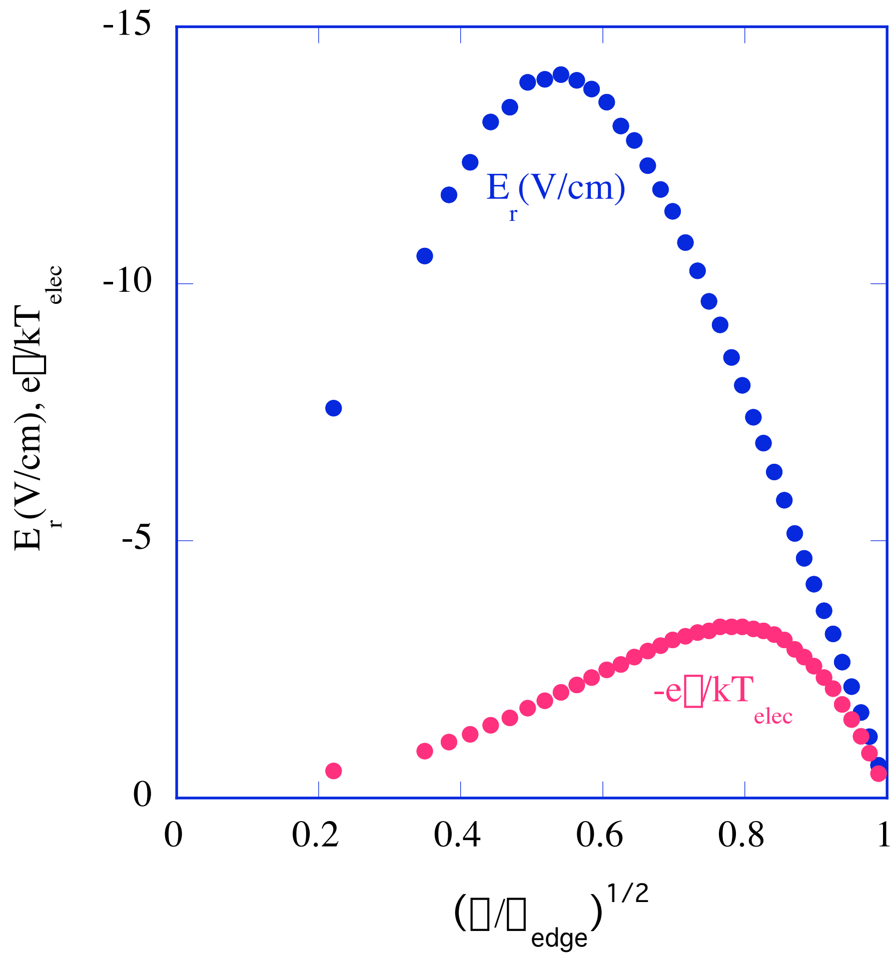
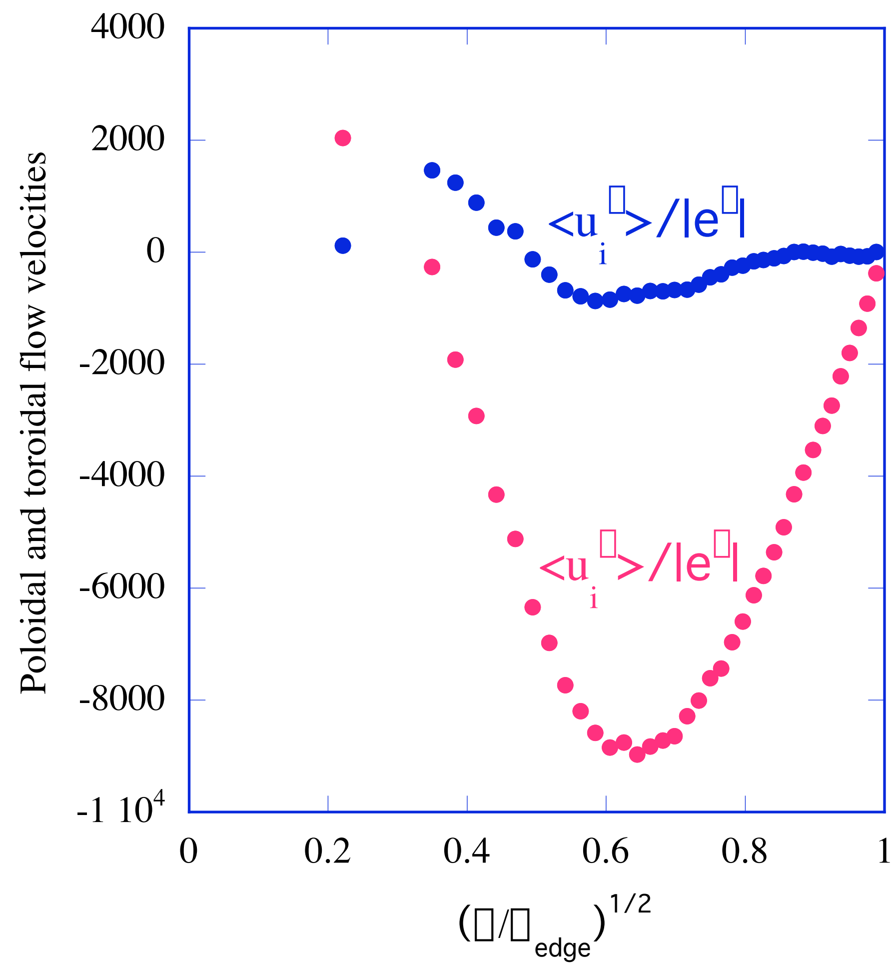
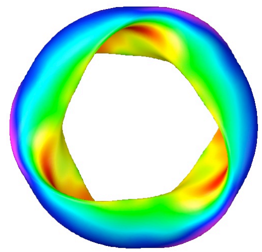




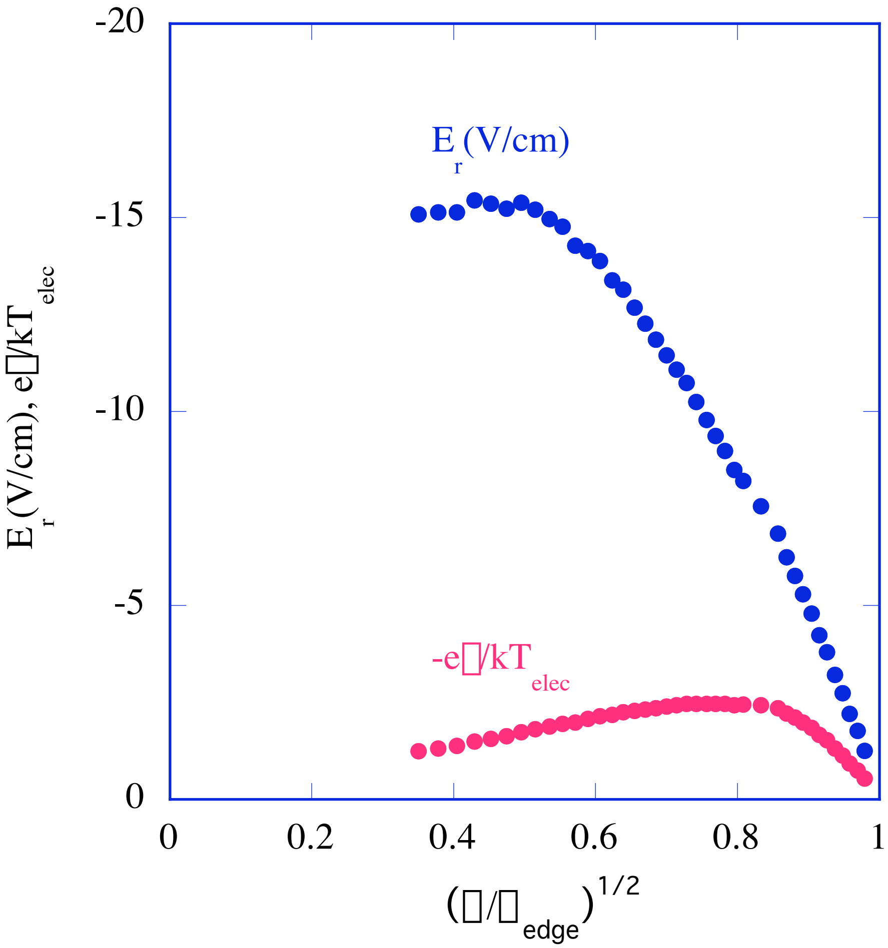
NCSX

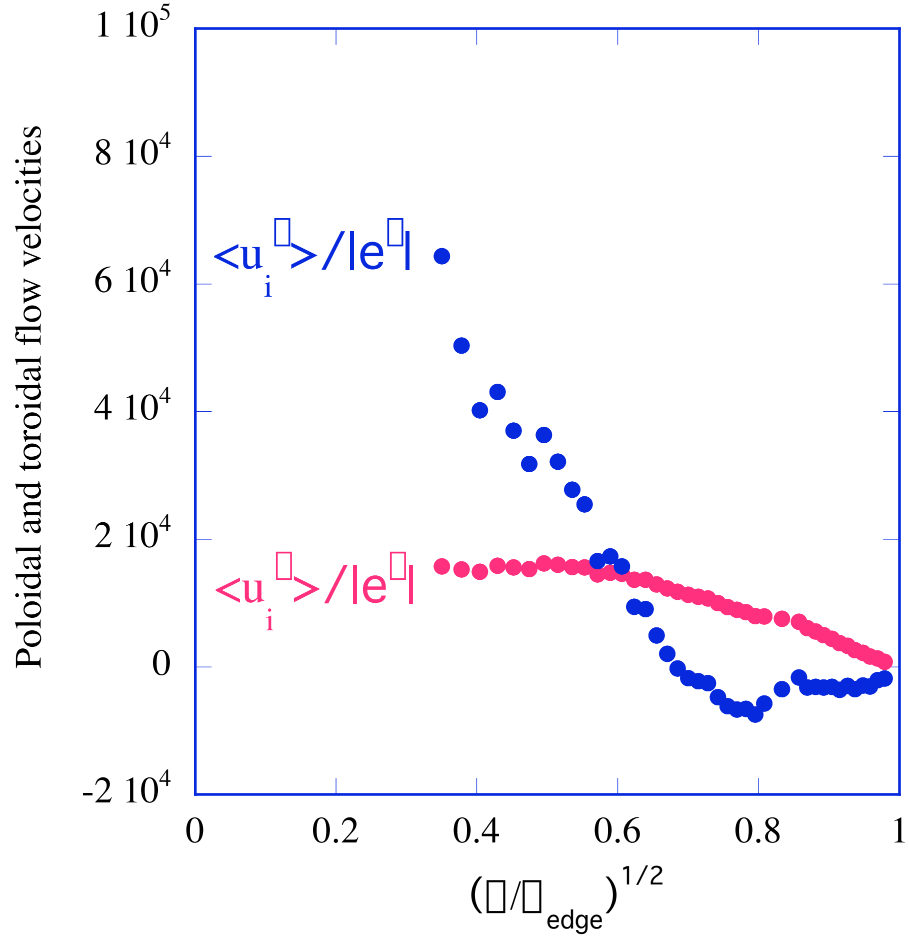
electric field and flow velocities



Toroidal flow dominant in center, poloidal flow at edge

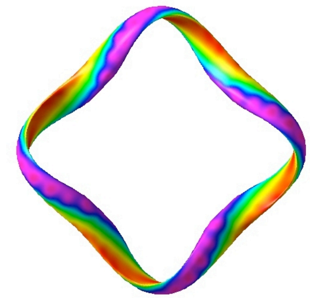
$n(0) = 8.3 \times 10^{19}\ m^{-3}$, $T_e(0) = 0.53$ keV, $T_i(0) = 0.38$ keV



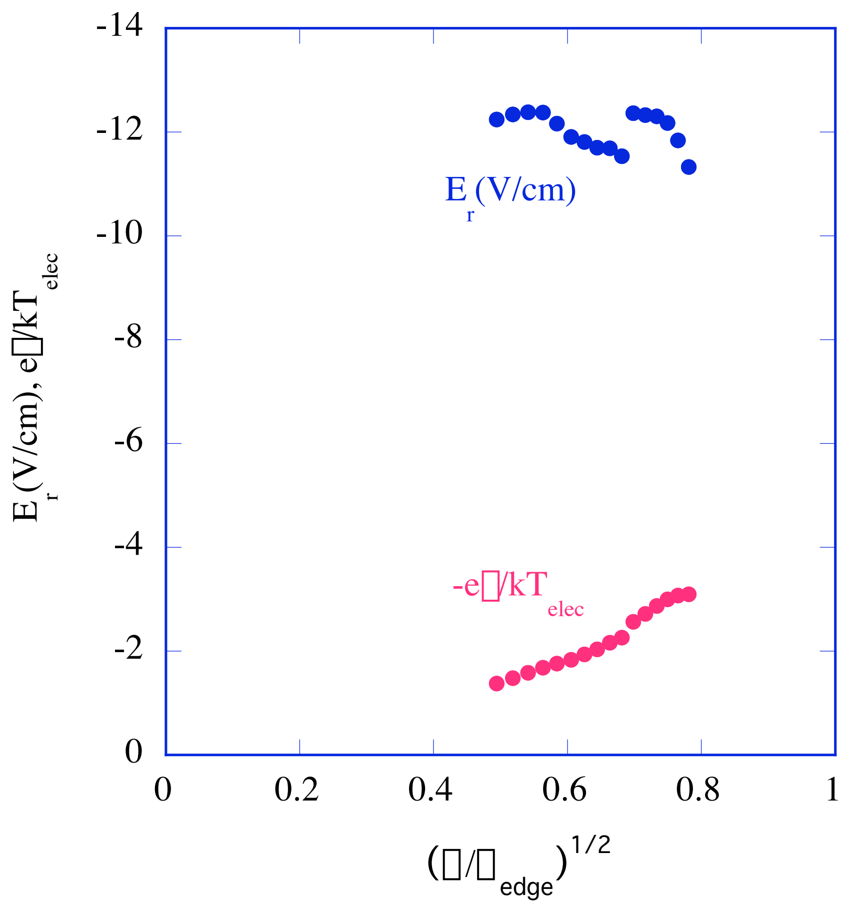


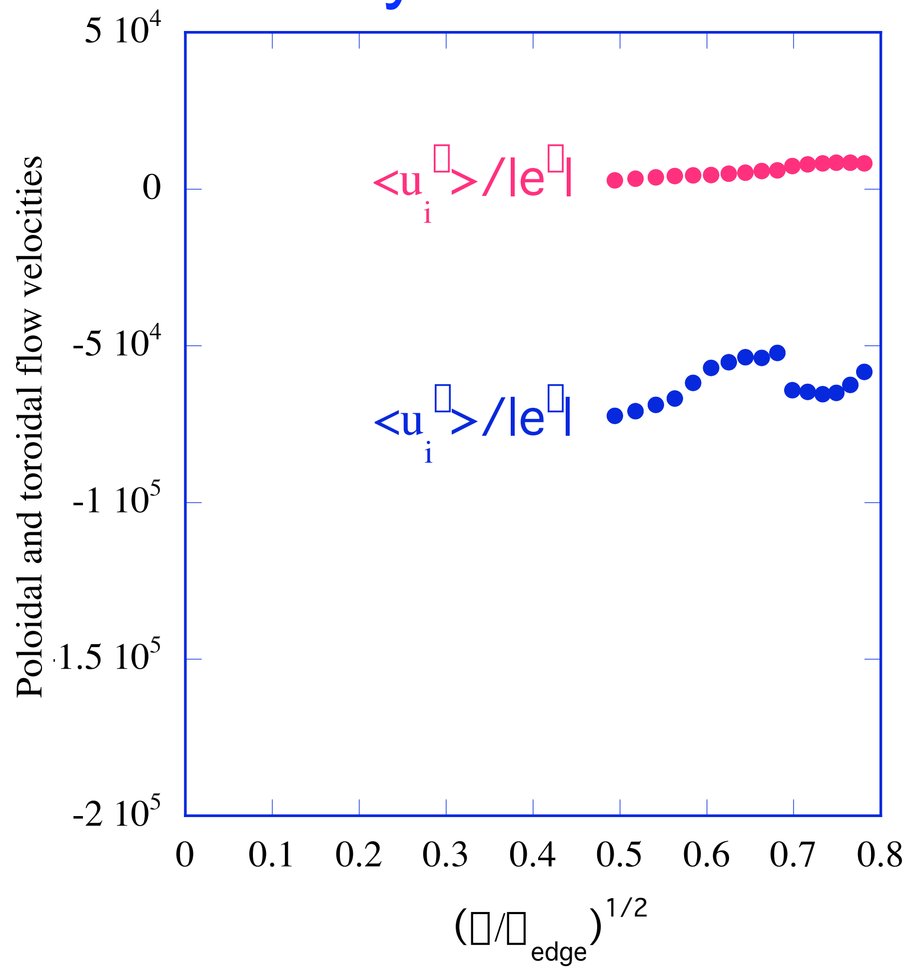
HSX

electric field and flow velocities
(radial scan incomplete at this point)



Flow follows helical |B| contours - mostly toroidal





Conclusions

- QPS shows continues to show strong dominance of poloidal flows
  - Expected efficiency for turbulence suppression due to
    - Extended nature of turbulent eddies along magnetic field lines (poloidal flow shear more effective than toroidal flow)
    - Weak coupling to toroidal flow generation (i.e., momentum source is not expended on driving toroidal flows)
- Future development of moments method
  - Continued refinement of transport coefficient calculation and connection formulas
  - Parallelization of DKES over electric field parameter
    - Will speed up turn-around on different configurations
  - Multi-ion species
    - Impurity flow velocities
    - Impurity accumulation studies
  - neutral flow damping
  - Study the multiple electric field roots and their stability
  - Bootstrap current evaluation/benchmark
  - External flow drive (bias electrodes, RF flow drive, beams)