TRANSPORT PHYSICS ISSUES OF LOW ASPECT RATIO STELLARATORS

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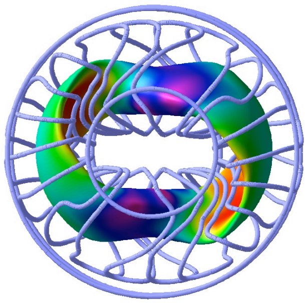
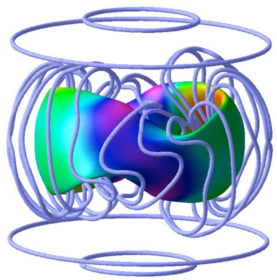




- QPS is a very low aspect ratio (A = 2.7) Quasi-poloidal stellarator that will test:
  - Equilibrium robustness at low A
  - Neoclassical and anomalous transport
  - Stability limits up to $\langle\beta\rangle$ = 2.5%
  - Bootstrap current effects
  - Reduced poloidal viscosity effects on shear flow transport reduction
  - and Configurational flexibility
- Design parameters: $\langle R_0 \rangle$ = 0.9 m, $\langle a \rangle$ = 0.33 m, $\langle B \rangle$ = 1 T ± 0.2T for 1sec, $I_p \leq$ 150 kA, P_{ECH} = 0.6-1.2 Mw, P_{ICH} = 1-3 Mw
- Recently Sugama, et al.[1] have adapted the moment method of Hirshman and Sigmar[2] to stellarator transport in a way that connects to transport coefficients provided by the DKES code
  - Uses fluid momentum balance equations and friction-flow relations that take into account momentum conservation
  - Viscosity coefficients are obtained from the drift kinetic equation
    - Uses l = 2 Legendre components of f (for which the test particle component of the collision term dominates over the field component)
    - Does not directly calculate Γ and Q from f because the field component of the collision operator is more significant for these moments
- Provides:
  - A way to assess viscosities in low aspect ratio quasi-symmetric devices
  - Momentum conserving corrections to DKES-based bootstrap currents, particle and energy flows.

[1] H. Sugama, S. Nishimura, Phys. Plasmas **9**, 4637 (2002).
[2] S. P. Hirshman, D. J. Sigmar, Nuclear Fusion **21**, 1079 (1981).

QPS viscosities
[based on S. Sugama, S. Nishimura, Phys. Plasmas **9** 4637 (2002)]

Viscous Forces
$$\begin{bmatrix} \langle \vec{B}_P \cdot (\vec{\nabla} \cdot \Pi) \rangle \\ \langle \vec{B}_T \cdot (\vec{\nabla} \cdot \Pi) \rangle \end{bmatrix} = \begin{bmatrix} M_{1PP} & M_{1PT} \\ M_{1PT} & M_{1TT} \end{bmatrix} \begin{bmatrix} \langle u^{\theta} \rangle / \chi' \\ \langle u^{\zeta} \rangle / \psi' \end{bmatrix}$$

where: Π = viscous stress tensor, u^{θ}, u^{ζ} = contravariant poloidal/toroidal flow velocities
(the heat flux terms in the above equation have not been indicated for simplicity)

$$M_{1PP},\ M_{1PT},\ M_{1TT} = \frac{2n}{\sqrt{\pi}} \int_0^{\infty} dK \sqrt{K} e^{-K} \left(K - \frac{5}{2}\right) [M_{PP}(K),\ M_{PT}(K),\ M_{TT}(K)]$$

where $K = m\mathrm{v}^2/2kT$ and

$$\begin{bmatrix} M_{PP} & M_{PT} \\ M_{PT} & M_{TT} \end{bmatrix} = \frac{4\pi^2}{V'} \begin{bmatrix} \chi' B_\zeta/\langle B^2 \rangle & -\frac{e}{c}\chi'\psi'/\chi' \\ \psi' B_\theta/\langle B^2 \rangle & \frac{e}{c}\psi'\chi'/\psi' \end{bmatrix} \begin{bmatrix} M & N \\ N & L \end{bmatrix} \begin{bmatrix} \chi' B_\zeta/\langle B^2 \rangle & \psi' B_\theta/\langle B^2 \rangle \\ -\frac{e}{c}\chi'\psi'/\chi' & \frac{e}{c}\psi'\chi'/\psi' \end{bmatrix}$$

We choose the following normalizations (following Sugama, et al.) for the viscosities:

$$M^* = \frac{M}{m\mathrm{v}_T K^{3/2}} = \frac{(\nu/\mathrm{v}) D_{33}^*}{1 - \frac{3}{2}\frac{\nu}{\mathrm{v}} D_{33}^* / \langle B^2 \rangle} \qquad L^* = \left(\frac{e}{c}\right)^2 \frac{L}{m\mathrm{v}_T K^{3/2}} = D_{11}^* - \frac{2}{3}\frac{\nu}{\mathrm{v}} \langle \tilde{U}^2 \rangle \frac{\frac{3}{2}\frac{\nu}{\mathrm{v}} (D_{13}^*)^2 / \langle B^2 \rangle}{1 - \frac{3}{2}\frac{\nu}{\mathrm{v}} D_{33}^* / \langle B^2 \rangle}$$

$$N^* = \frac{e}{c} \frac{N}{m\mathrm{v}_T K^{3/2}} = \frac{(\nu/\mathrm{v}) D_{13}^*}{1 - \frac{3}{2}\frac{\nu}{\mathrm{v}} D_{33}^* / \langle B^2 \rangle}$$

where the normalized transport coefficients below are in the form generated by DKES:

$$D_{11}^* = D_{11} / \left[\frac{1}{2}\mathrm{v}_T \left(\frac{B\mathrm{v}_T}{\Omega}\right)^2 K^{3/2}\right], \quad D_{13}^* = D_{13} / \left[\frac{1}{2}\mathrm{v}_T \left(\frac{B\mathrm{v}_T}{\Omega}\right) K\right], \quad D_{33}^* = D_{33} / \left[\frac{1}{2}\mathrm{v}_T K^{1/2}\right]$$

The monoenergetic viscosities M_{pp}, M_{pt}, M_{tt} are then normalized as:

$$\left(M_{PP},\ M_{PT},\ M_{TT}\right) = 4\pi^2 m\mathrm{v}_T K^{3/2} \left(M'_{PP},\ M'_{PT},\ M'_{TT}\right)$$

Expanding the above matrix products leads to:

$$M'_{PP} = \frac{(\psi'\chi')^2}{V'} \left\{ \frac{[B_\zeta/\langle B^2 \rangle]^2}{(\chi')^2} M^* - \frac{2B_\zeta/\langle B^2 \rangle}{\chi'} N^* + L^* \right\}$$

$$M'_{PT} = \frac{(\psi'\chi')^2}{V'} \left\{ \frac{B_\zeta B_\theta/\langle B^2 \rangle}{\chi'\psi'} M^* - \frac{2B_\zeta/\langle B^2 \rangle}{\chi'} N^* - L^* \right\}$$

$$M'_{TT} = \frac{(\psi'\chi')^2}{V'} \left\{ \frac{[B_\theta/\langle B^2 \rangle]^2}{(\psi')^2} M^* - \frac{2B_\theta/\langle B^2 \rangle}{\psi'} N^* + L^* \right\}$$

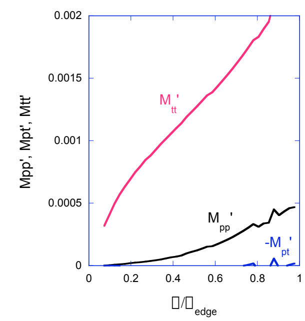
Similarly, the parallel viscosity and radial transport flows are given by:

$$\begin{bmatrix} \langle \vec{B} \cdot (\vec{\nabla} \cdot \Pi) \rangle \\ \\ \Gamma \end{bmatrix} = \begin{bmatrix} M_1 & N_1 \\ N_1 & L_1 \end{bmatrix} \begin{bmatrix} \dfrac{\langle u_{\|} B \rangle}{\langle B^2 \rangle} \\ \\ -\dfrac{1}{n}\dfrac{\partial p}{\partial s} - e\dfrac{\partial \Phi}{\partial s} \end{bmatrix}$$

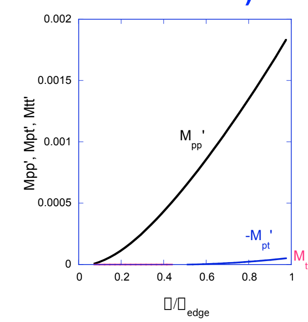
where

$$(M_1,\ N_1,\ L_1) = \frac{2nm\mathrm{v}_T}{\sqrt{\pi}} \int_0^{\infty} dK \sqrt{K} K^{3/2} \left[M^*(K),\ \left(\frac{c}{e}\right) N^*(K),\ \left(\frac{c}{e}\right)^2 L^*(K)\right]$$

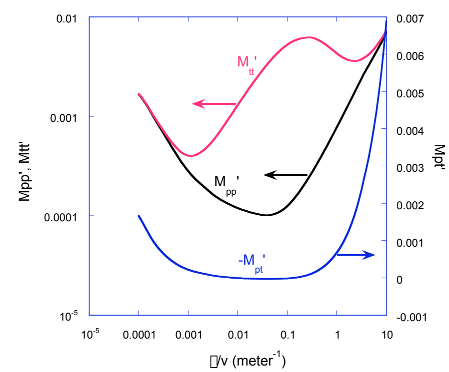
QPS Viscosities (toroidal damping dominates)



Equivalent Tokamak Viscosities (poloidal damping dominates)



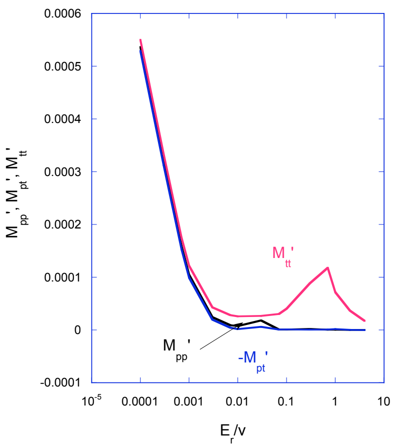
Collsionality dependence of QPS viscosity



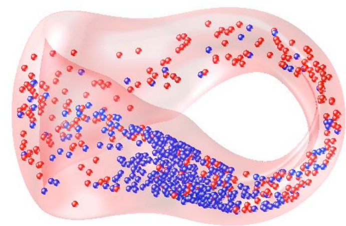
Monte Carlo transport studies using the DELTA5D model

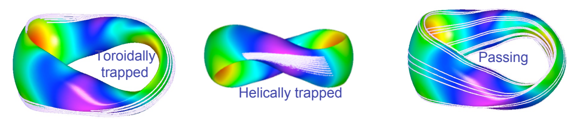
Alfvén Gap structure using STELLGAP code

E_r dependence of QPS viscosity

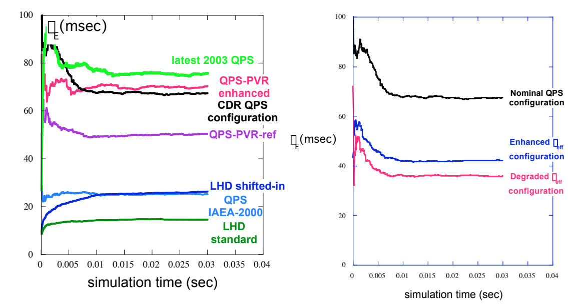


- QPS configurations provide unique features for viscous flow damping physics
  - Poloidal viscosity << toroidal viscosity
  - Viscosity anisotropy orthogonal to that of tokamaks
  - Less damping toward poloidal flows and control of radial viscous profiles through coil optimizations could allow better access to enhanced confinement regimes





Global ion confinement times for ICH parameters between devices (all at same R_0) and for QPS coil current variations



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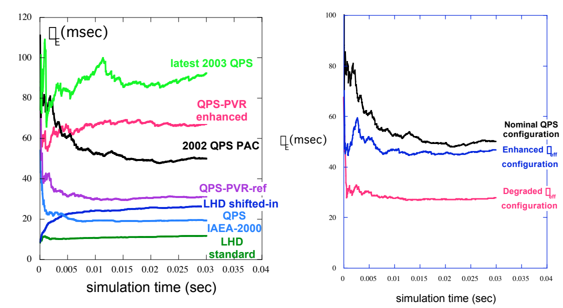

Stellarator Alfvén Couplings

Alfvén coupling condition: $k_{\|,m,n} = -k_{\|,(m+\mu),(n+\nu N_{fp})}$

μ, ν = integers

$$n - m\iota = -\left(n + \nu N_{fp} - (m+\mu)\iota\right)$$

$$\iota = \frac{2n + \nu N_{fp}}{2m+\mu} \qquad \omega = \frac{v_A}{R}\,\frac{\mu n - \nu m N_{fp}}{2m+\mu}$$

- GAE (global Alfvén mode): $\nu = 0,\ \mu = 0$
- TAE (toroidal Alfvén mode): $\nu = 0,\ \mu = \pm 1$
- EAE (elliptical Alfvén mode): $\nu = 0,\ \mu = \pm 2$
- NAE (noncircular Alfvén mode): $\nu = 0,\ |\mu| > 2$
- MAE (mirror Alfvén mode): $\nu = 1,\ \mu = 0$
- HAE (helical Alfvén mode): $\nu = 1,\ \mu \neq 0$