Equilibrium and MHD Stability of the QPS Configuration

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Equilibrium of the QPS Configuration

- MHD Equilibrium
  - A low aspect ratio configuration with rotational transform $\iota(0)\sim0.2$ to $\iota(a)\sim0.4$
- Bootstrap Current
  - Bootstrap consistent equilibria for $\langle\beta\rangle=0$ to 2%
- Ohmic Current
  - Configurations with Ohmic current profiles have weak shear iota profiles

MHD Stability for Free and Fixed Boundary Calculations

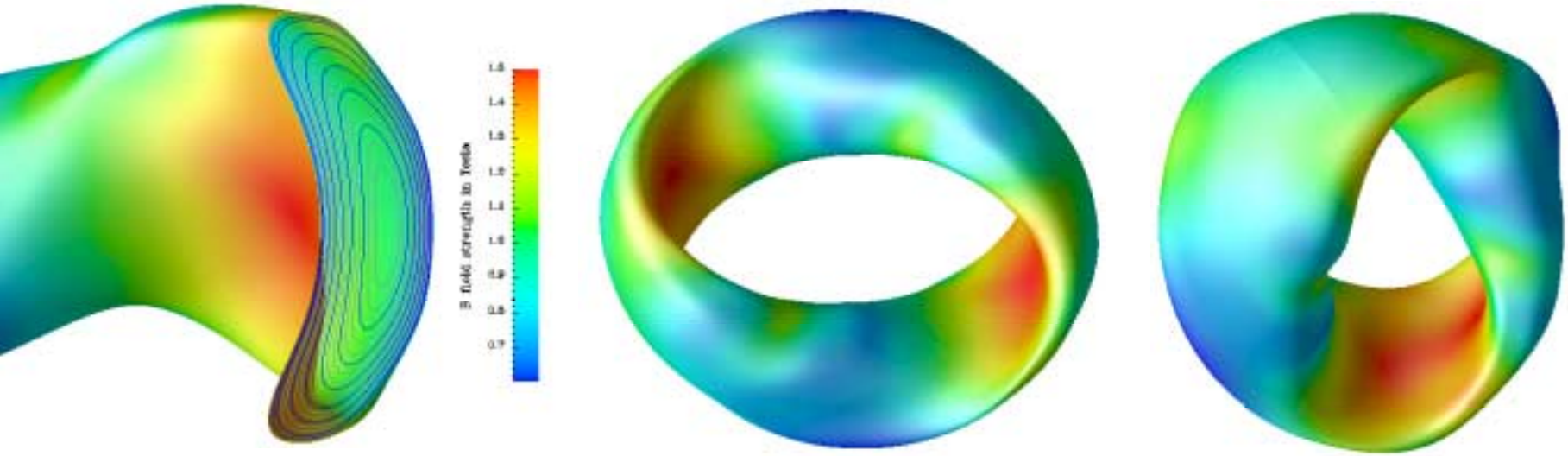
- Mercier Stability
  - Magnetic well provides stability except at isolated resonances
- Ballooning Stability
  - Sets the stability β-limits for QPS $\langle\beta\rangle$=2.5% fixed; $\langle\beta\rangle$=2.2% free
- Kink and Vertical Stability
  - Small amount of plasma current results in good kink and vertical stability

MHD Equilibrium

- The 3D equilibrium code VMEC obtains equilibria by minimizing the plasma potential energy: $\int (B^2/2 + p)dV$
  - VMEC assumes nested flux surfaces exists (does not allow for the existence of magnetic islands)
  - Good approximations of real equilibria when magnetic islands are small
  - Used to design every modern stellarator

MHD Equilibrium: Flux surfaces of the QPS reference configuration

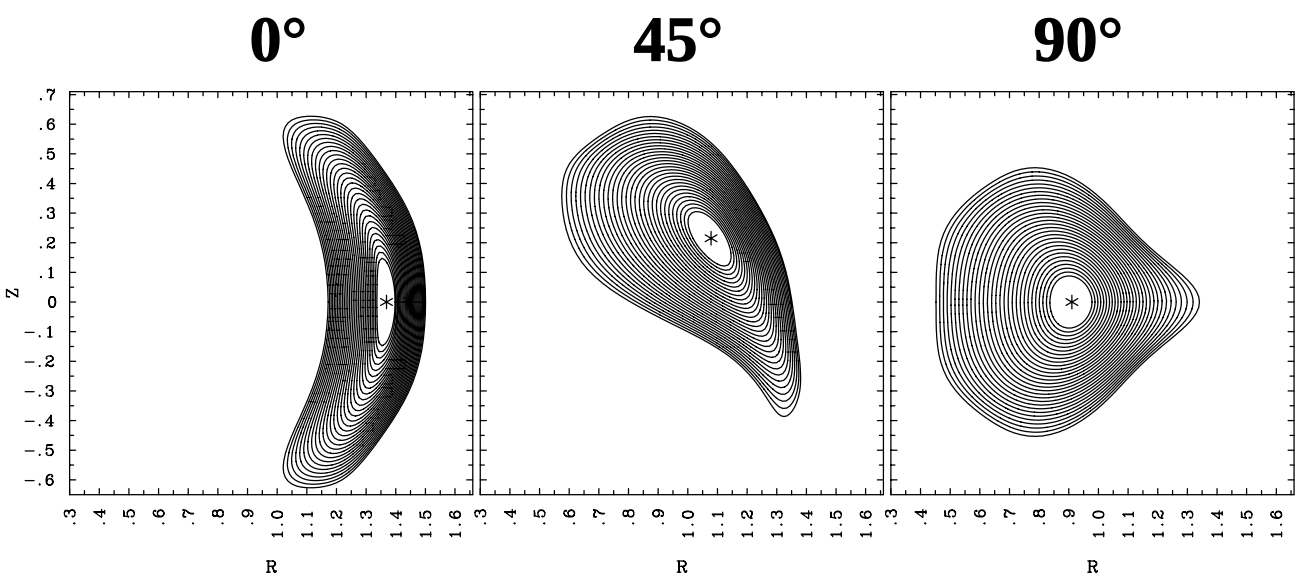
- LCFS & cross section, free-boundary QPS

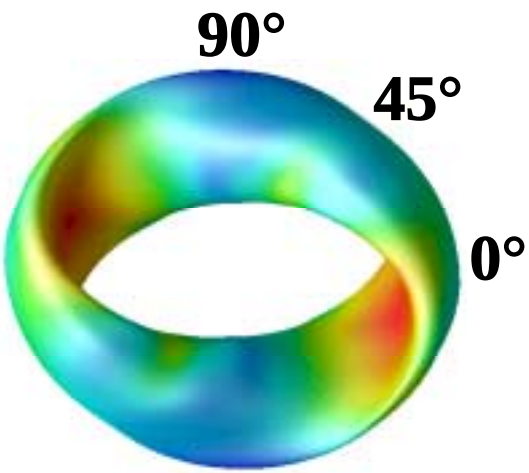


A = 2.6, **<β> = 2%,** **I_ϕ = 60 kA**

MHD Equilibrium: Cross Sections of the QPS reference configuration

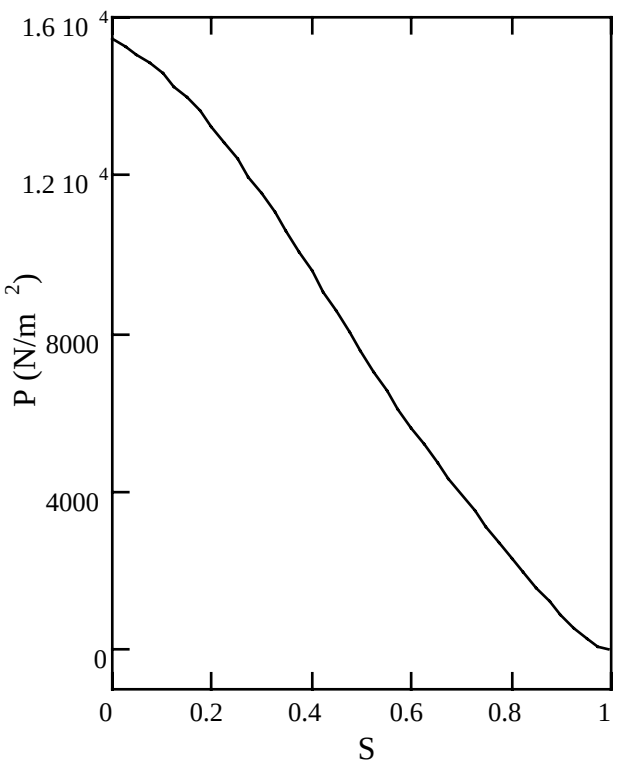
- Cross sections, free-boundary QPS

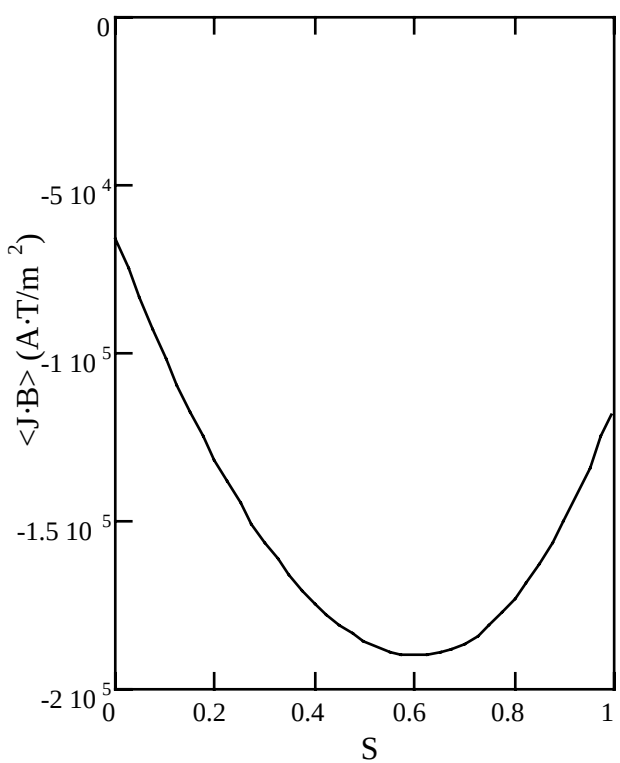


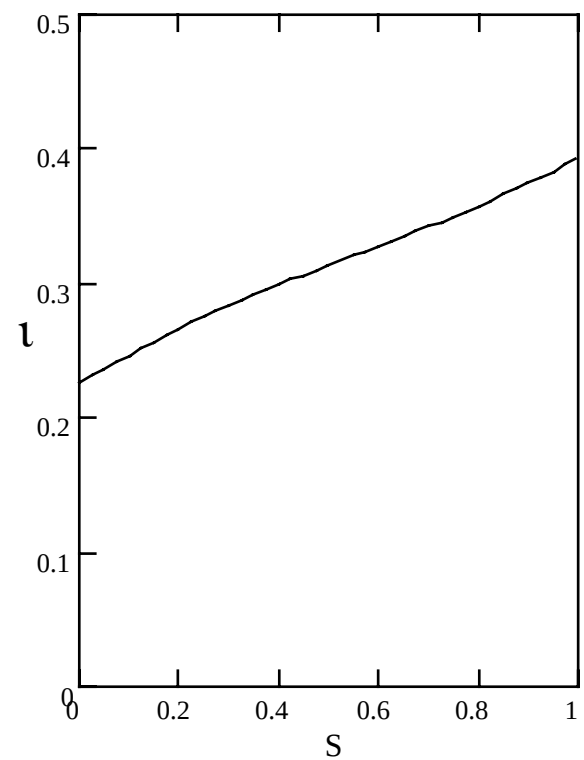


MHD Equilibrium: Profiles for the QPS reference configuration

- Pressure, current, and rotational transform





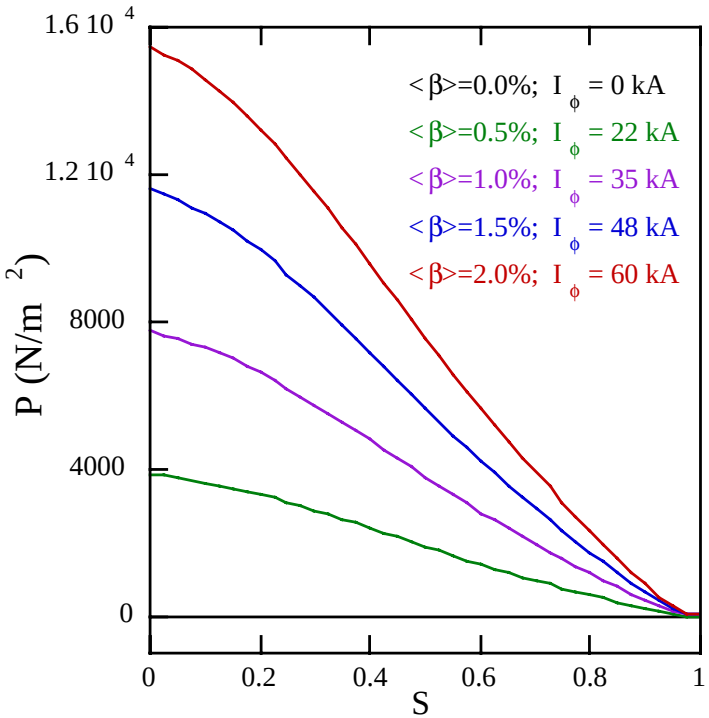


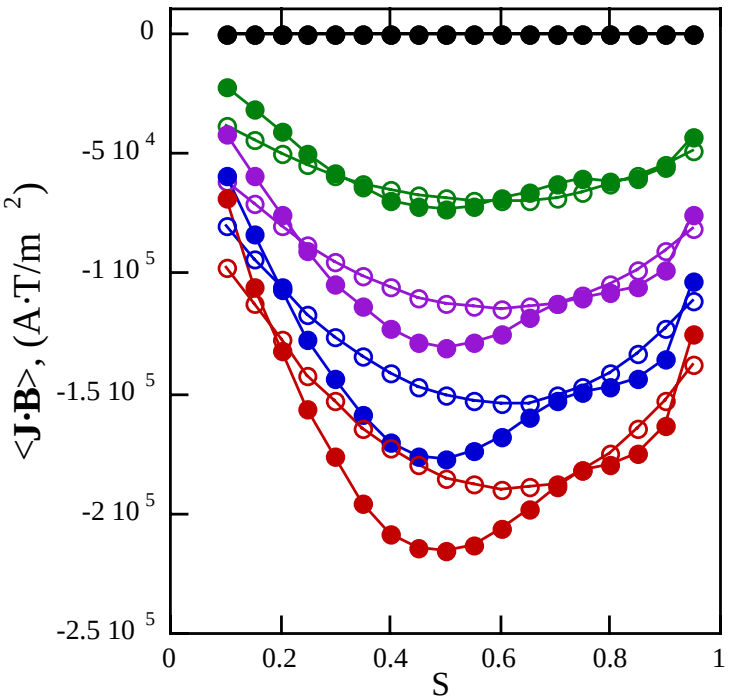
Bootstrap Current

- QPS reference configuration was designed for consistency of the equilibrium and bootstrap current profiles
  - (i.e., bootstrap current provides the majority of plasma current)
- Bootstrap current is calculated numerically based on a low-collisionality formulation

Bootstrap Current: Configurations with good bootstrap consistency for <β>=0-2%

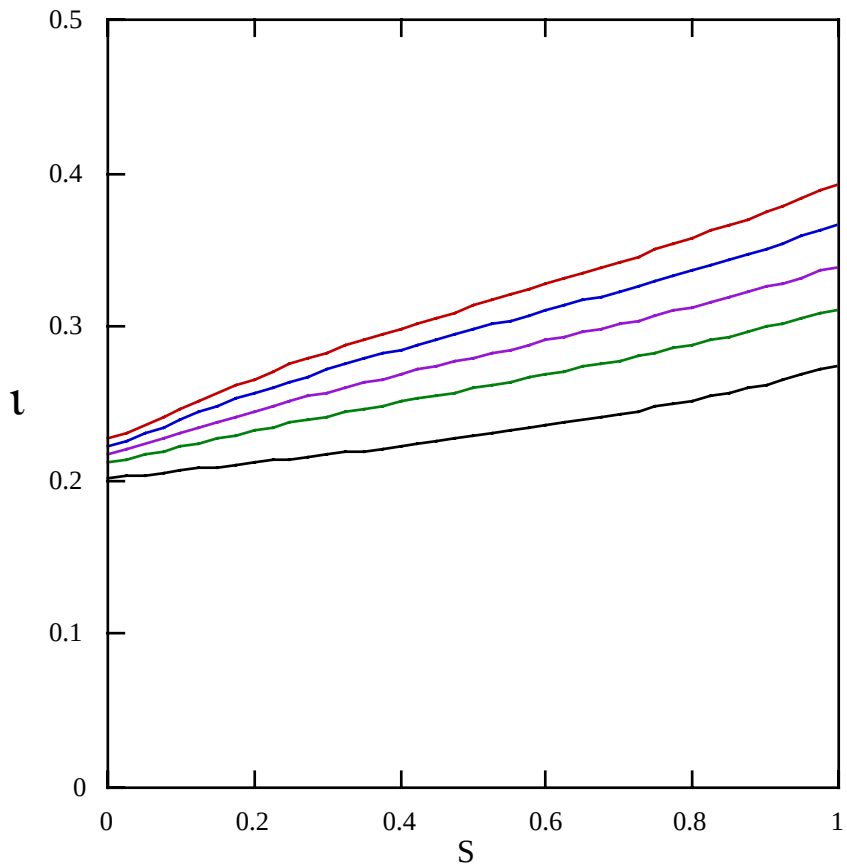
- Pressure and parallel current profiles, <β>=0-2%





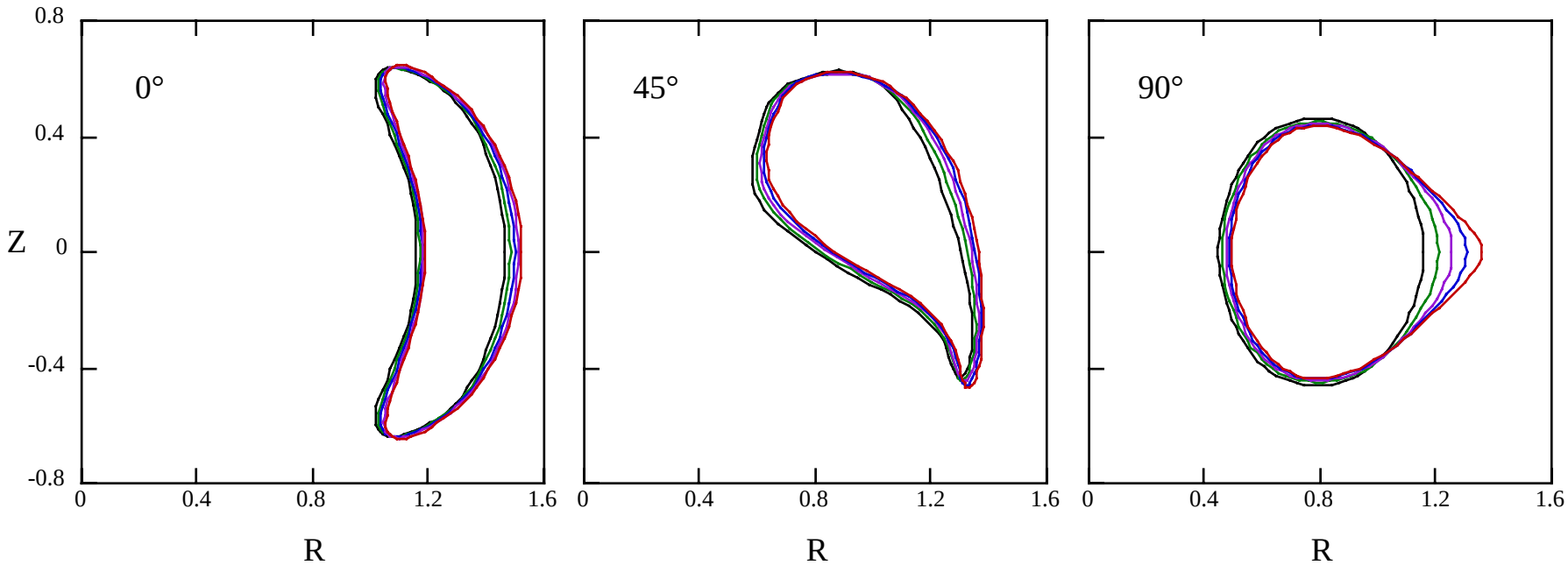
Bootstrap Current: Shear in rotational transform increases as β, I_{BS} increase

- Rotational transform profiles: <β>=0,0.5,1,1.5,2%



Bootstrap Current: Plasma shifts outward & triangularity goes up as β, I_{BS} increase

- Outer flux surfaces: <β> = 0, 0.5, 1, 1.5, 2%

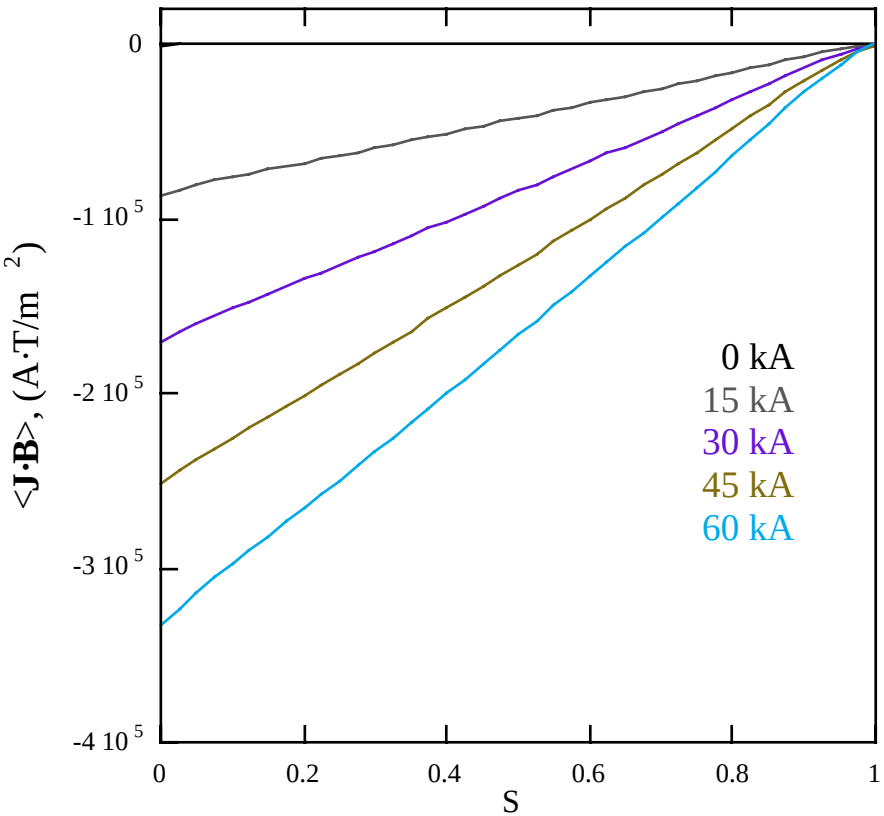


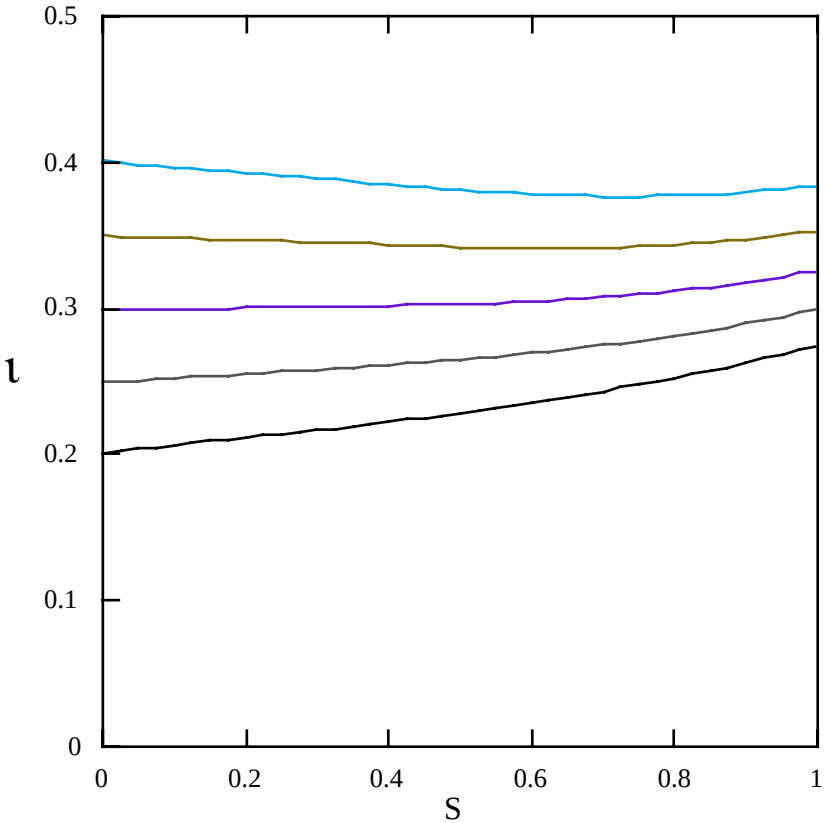
Ohmic Current

- QPS device is designed to be able to operate with Ohmic current
- Chose a simple Ohmic profile (peaked on axis) and scanned the total toroidal current: 0, 15, 30, 45, 60 kA
- Ohmic current produces flat rotational transform profiles in QPS; even weakly reversed configurations

Ohmic Current: Induces a flat rotational transform

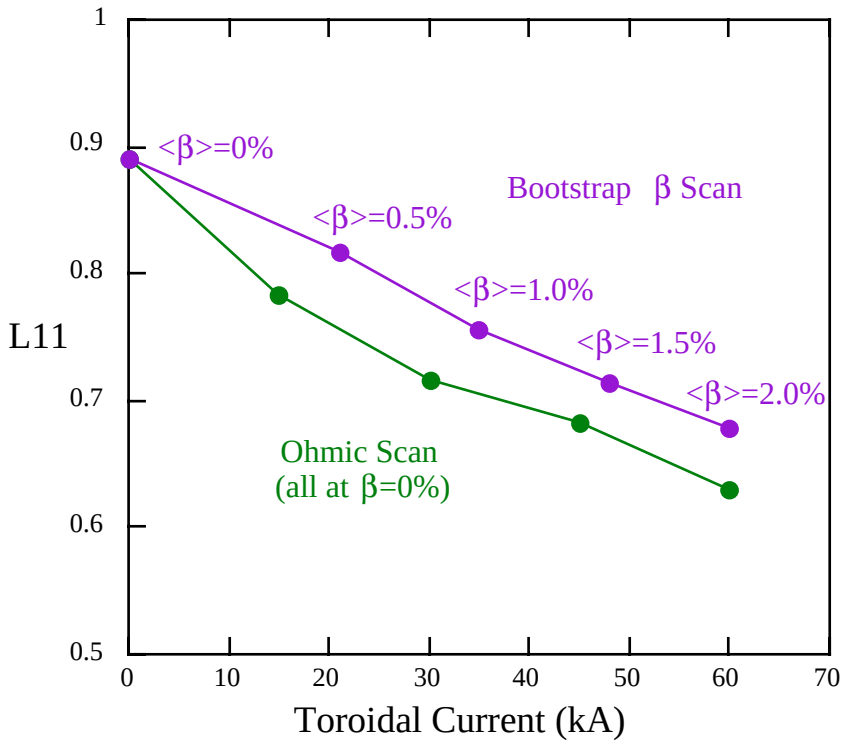
- Parallel current and rotational transform, Ohmic current scan





Ohmic Current: Similar to bootstrap current in effect on transport

- DKES (Drift Kinetic Equation Solver) radial transport coefficients on S=0.7 surface



MHD Stability

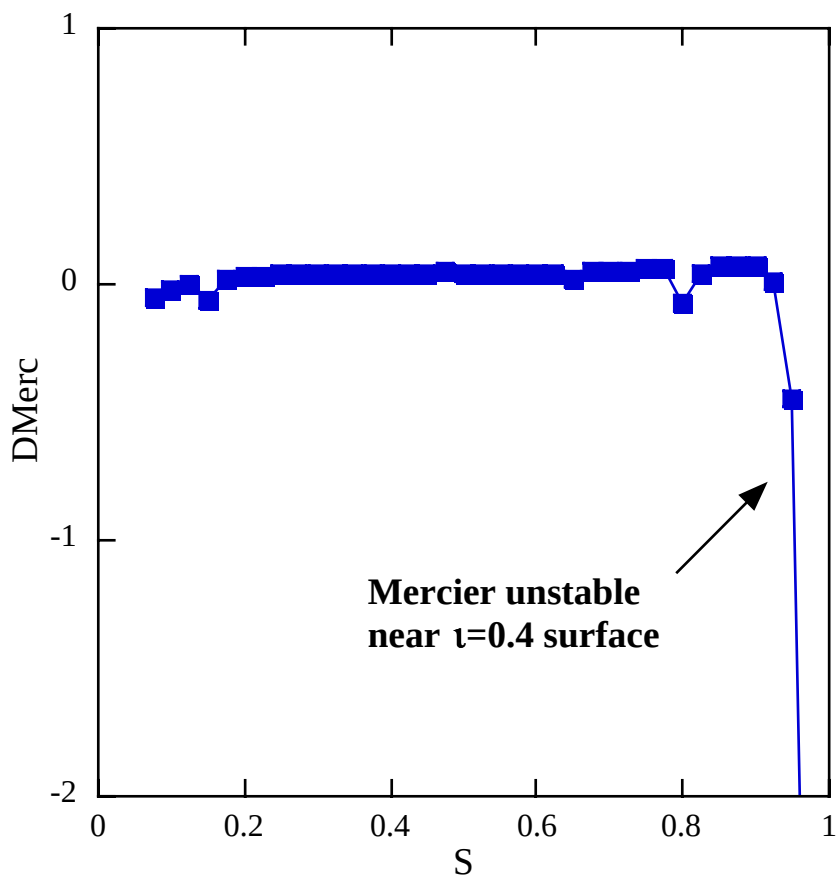
- What codes have been used to analyze ideal MHD stability?
- What sets the β stability limits in QPS?
- Mercier, ballooning, and kink/vertical stability

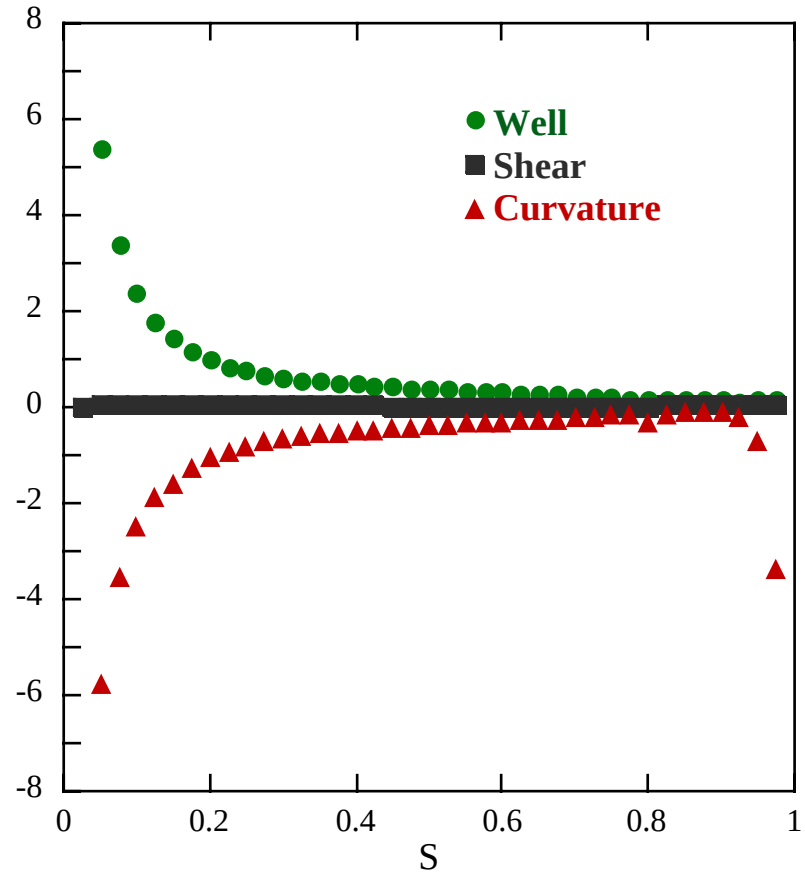
Mercier Stability

- The Mercier stability criterion is an indication of high-*n* interchange stability *(positive values indicate stability)*
- Evaluated directly in the VMEC code
- Magnetic well provides stability except at isolated resonances

Mercier Stability: A Magnetic Well Provides Stability Except at Isolated Resonances

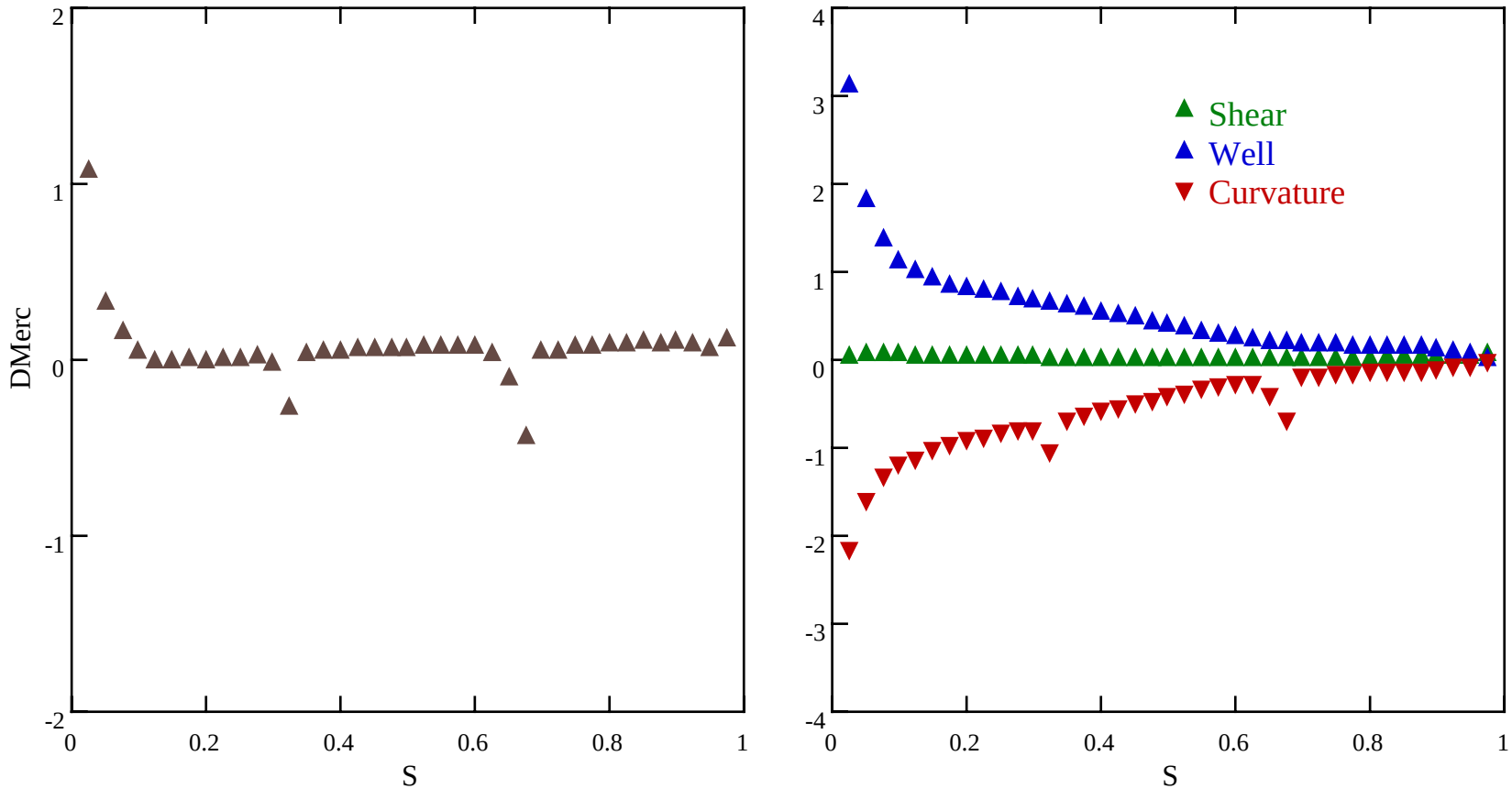
- **Mercier stability criteria for a fixed boundary case at <β>=2.5% (+ stable)**





Mercier Stability: A Magnetic Well Provides Stability Except at Isolated Resonances

- **Mercier stability criteria for a free boundary case at <β>=2.0% (+ stable)**

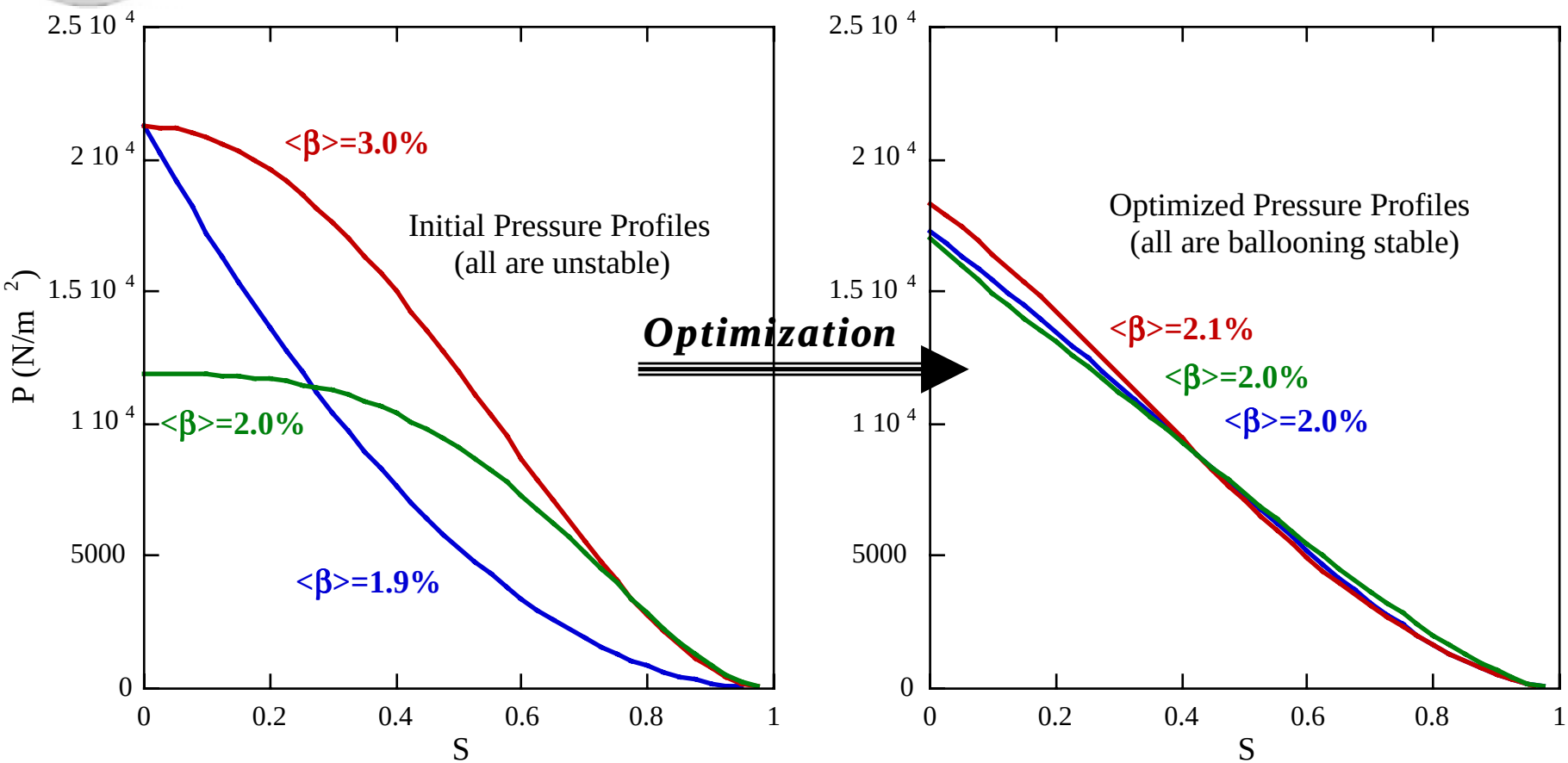


Ballooning Stability

- Ballooning stability analyzed with COBRA (**C**ode for **B**allooning **R**apid **A**nalysis) *[R. Sanchez, et al. J. Comp. Phys. 2000]*
  - Developed to meet the need for rapid ballooning analysis in the optimization
  - Latest version evaluates ballooning stability in VMEC coordinates (a few seconds to analyze stability on ~50 surfaces)

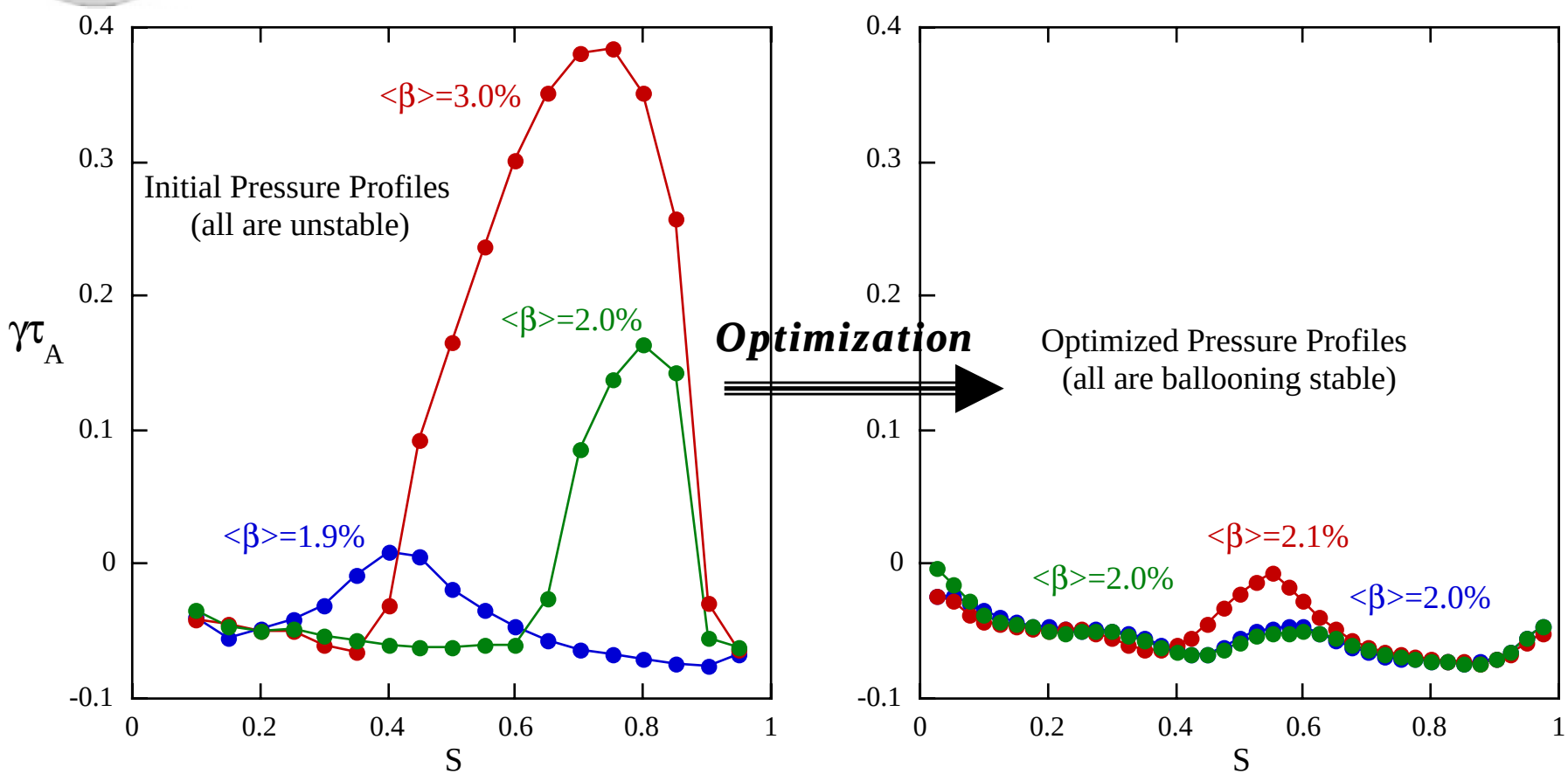
Ballooning Stability: β-limits improved by modifying the pressure profile

- **Pressure profiles optimized for ballooning stability at <β>=2% (fixed)**



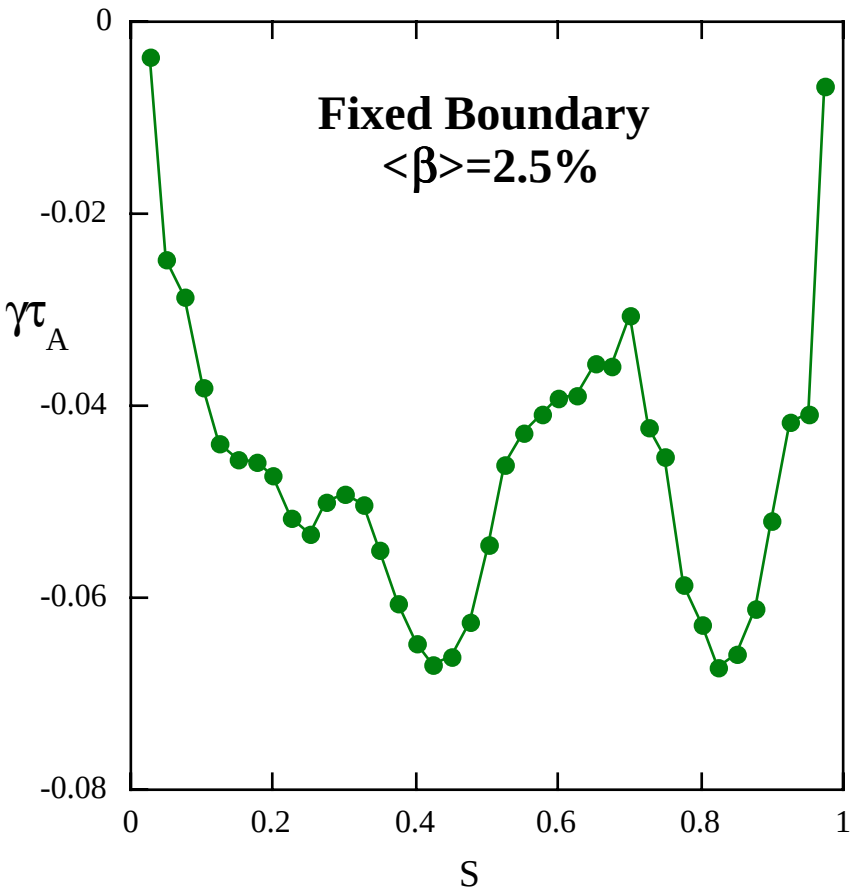
Ballooning Stability: β-limits improved by modifying the pressure profile

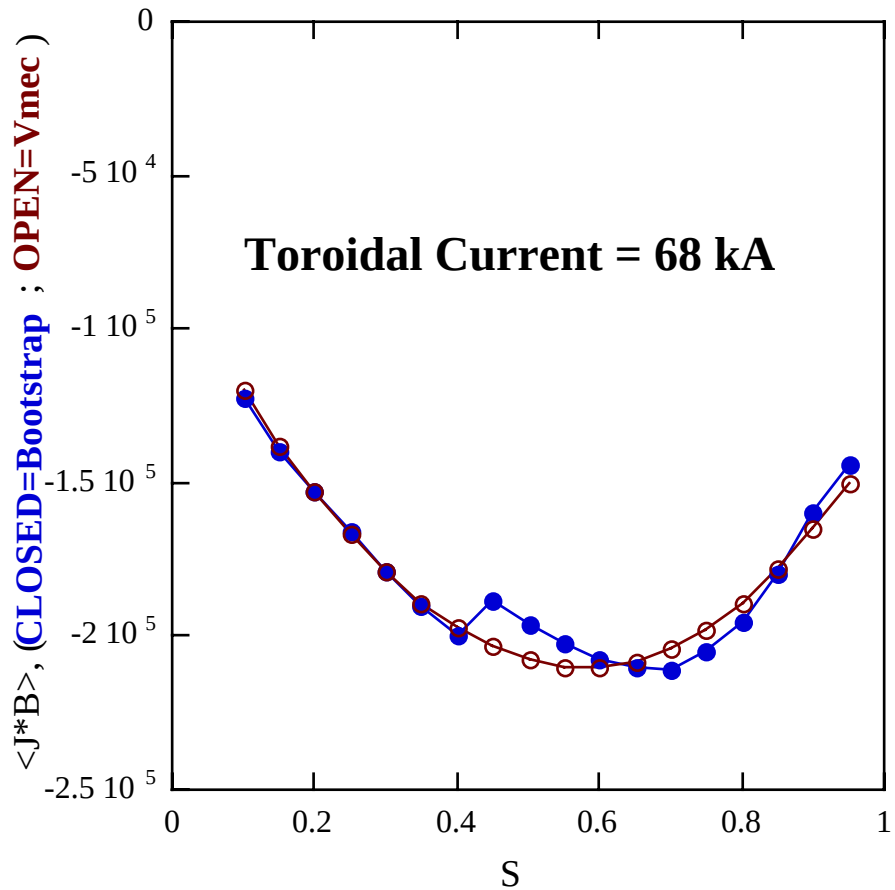
- **Ballooning growth rates for unoptimized & optimized cases (- stable)**



Ballooning Stability: Highest β Case - stable & bootstrap consistent at <β>=2.5%

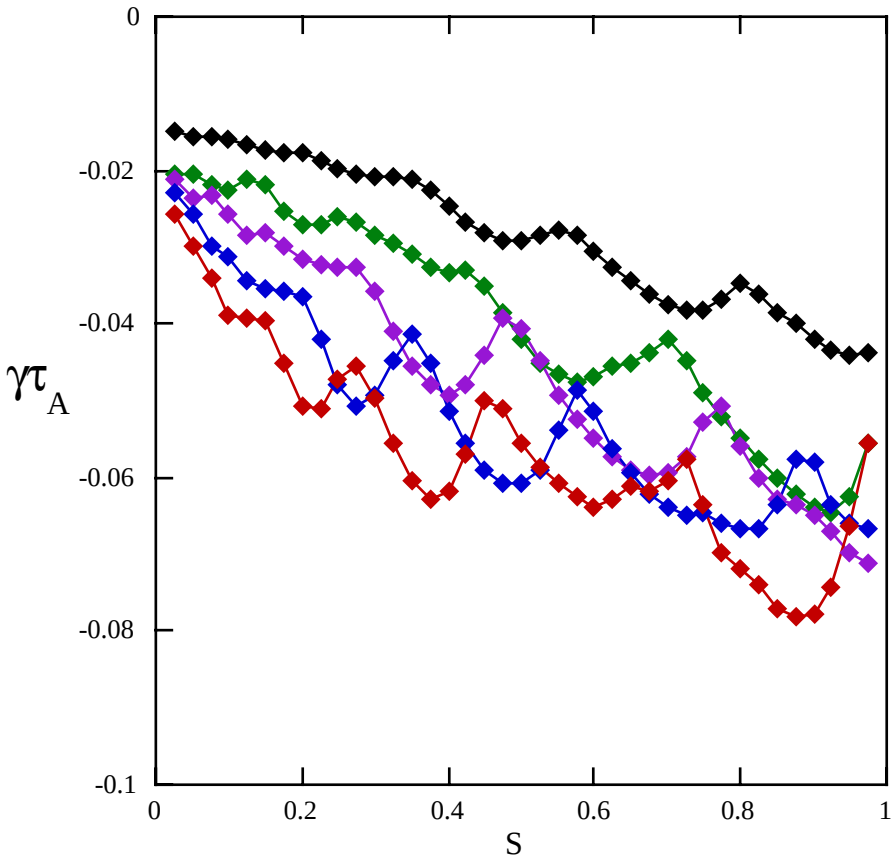
- **Ballooning growth rate & field-aligned current at <β>=2.5%, (fixed)**

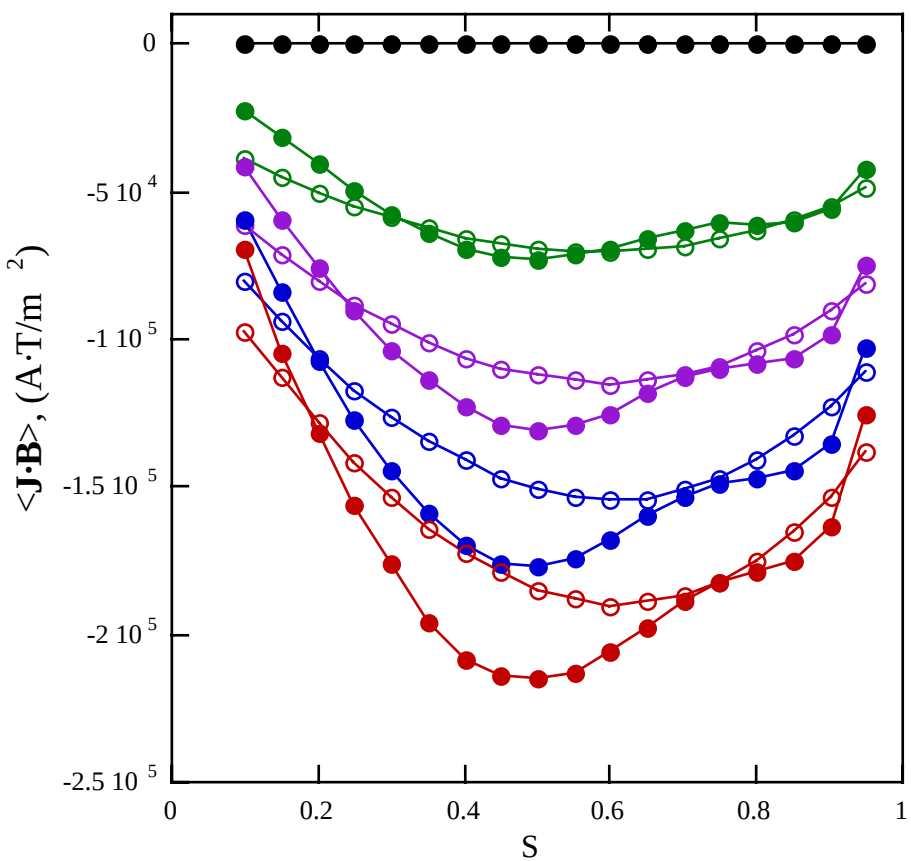




Ballooning Stability: Free boundary β-scan, stable & bootstrap consistent, $0 \le \langle\beta\rangle \le 2.0\%$

- **Ballooning growth rate & parallel current, free boundary β scan**





Vertical/Kink Stability

- Finite current in QPS configurations makes them potentially susceptible to low-*n* ideal MHD modes
- TERPSICHORE Code used to evaluate *n*=0 (vertical) and *n*=1 (kink) stability
  - All the cases stable to vertical modes at $\langle\beta\rangle \leq 5\%$ (both free and fixed boundary)
  - Free-boundary case unstable to kink mode at $\langle\beta\rangle \sim 5\%$

Kink & Vertical Stability: TERPSICHORE code used to test kink and vertical stability

- TERPSICHORE evaluates the MHD energy principle:

$$\omega^2 \delta W_k = \delta W_p + \delta W_{vac}$$

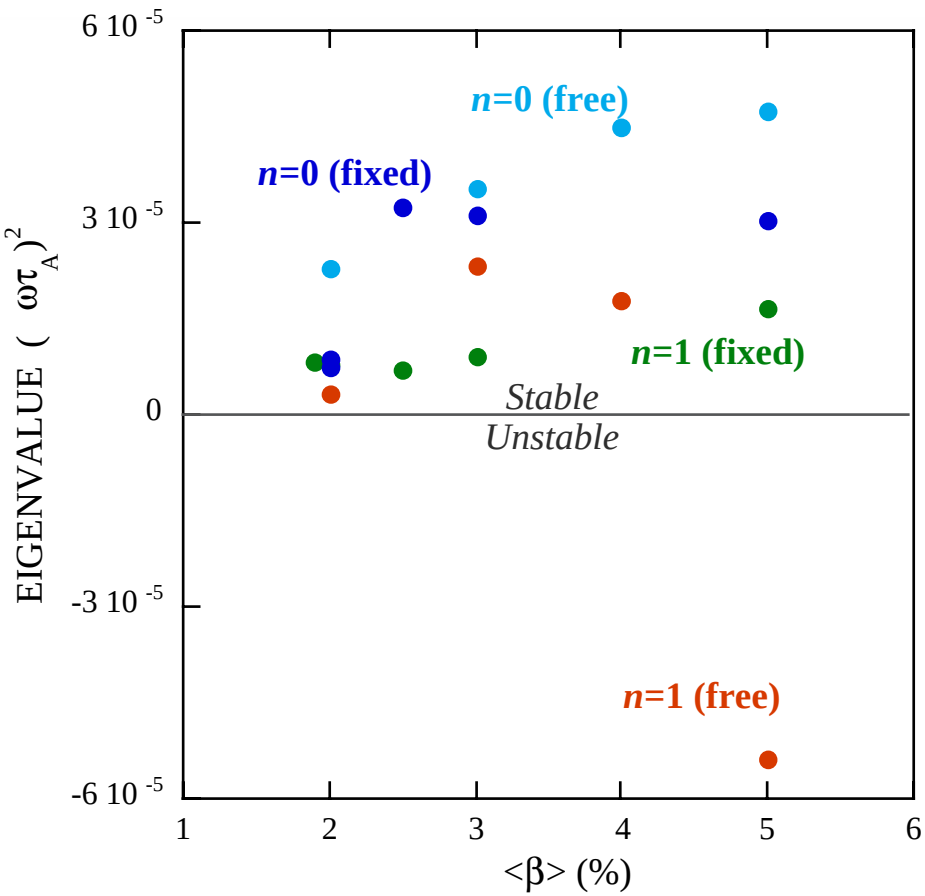
$$\delta W_p = \frac{1}{2} \int d^3x \left[\delta \mathbf{B}_\perp^2 + \left(\delta \mathbf{B}_{||} - \mathbf{B} \frac{\xi \cdot \nabla p}{B^2} \right)^2 \right.$$

$$\left. + \mathbf{j}_{||} \cdot \xi \times \delta \mathbf{B} - 2 \xi \cdot \nabla p \, \xi \cdot \kappa \right]$$

- Positive values of the eigenvalue, ω^2, imply stability

Kink & Vertical Stability: QPS reference configuration kink/vertical stable for $\langle\beta\rangle < 5\%$

- n=0 and n=1 eigenvalues for the gb4 cases tested



Summary

- QPS reference configuration is
  - a low-aspect ratio, current-carrying stellarator…
  - with a bootstrap consistent current profile…
  - also capable of running with Ohmic current
- QPS reference configuration meets the goal of ideal MHD stability at $\langle\beta\rangle \leq 2\%$ (free boundary)

Ongoing Work

- Investigate effects of a combined bootstrap and Ohmic current profile

  (e.g., configuration with 30 kA Ohmic + 30 kA bootstrap)
- Examine the effect of an edge pressure pedestal on MHD stability in QPS
- Explore link between the QPS reference configuration and the high-β reactor vision